



# CS-541

# Wireless Sensor Networks

## Lecture 9: Distributed In-network Processing for WSN

Spring Semester 2017-2018

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# Today's objectives

## Network signal processing types

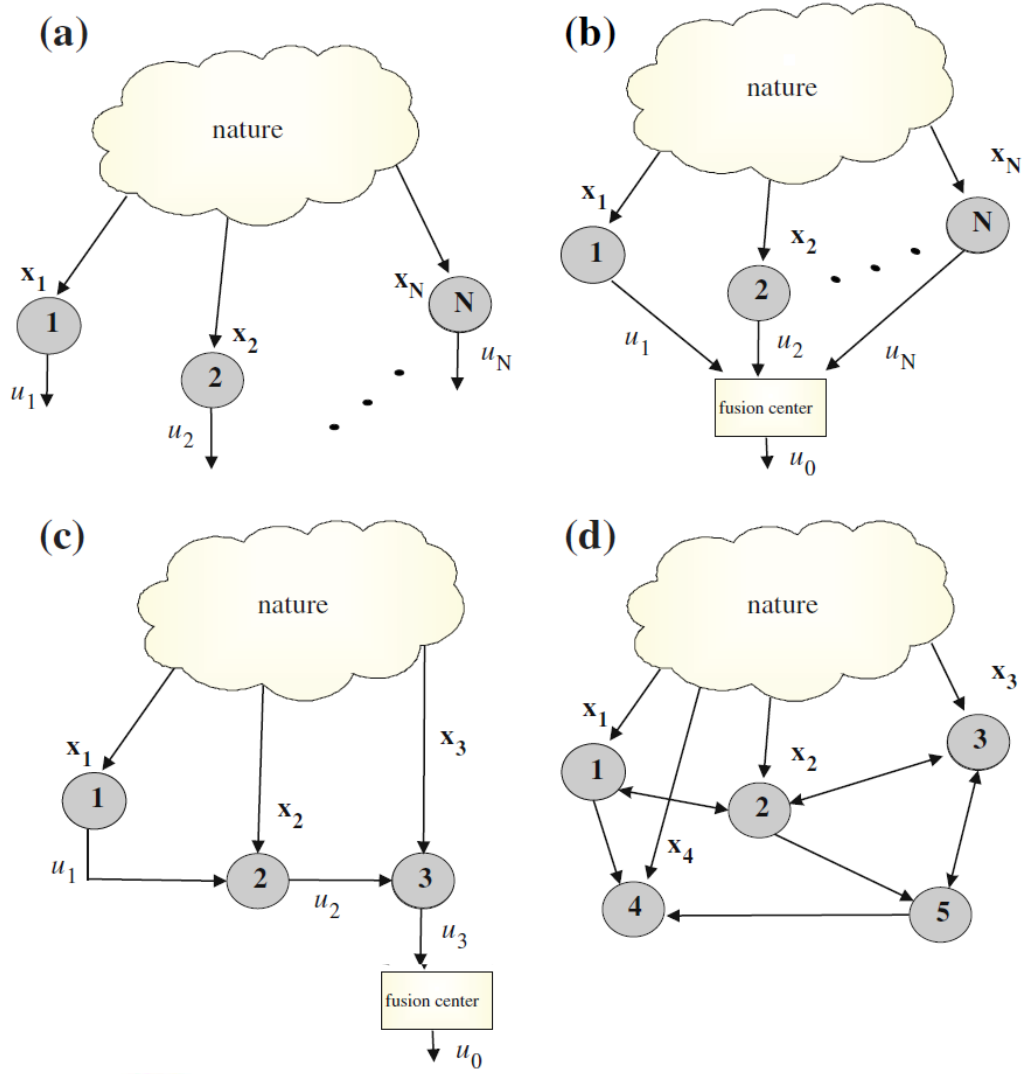
Centralized → Graph signal processing

Routing based → Network coding

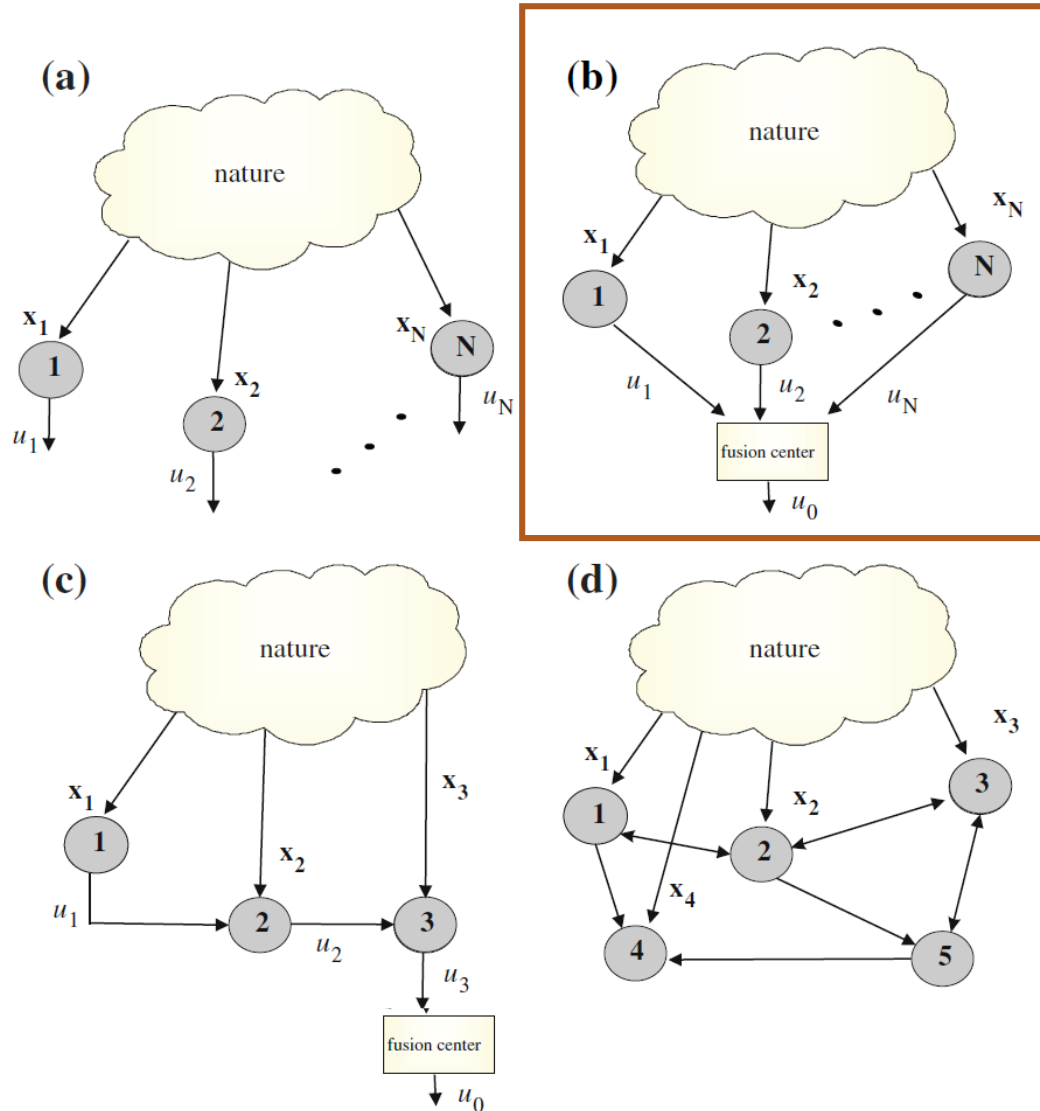
Distributed → Gossip



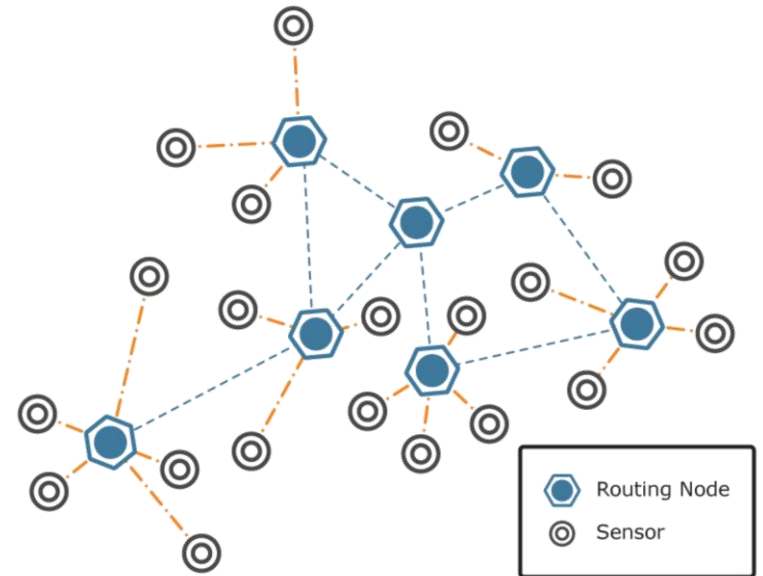
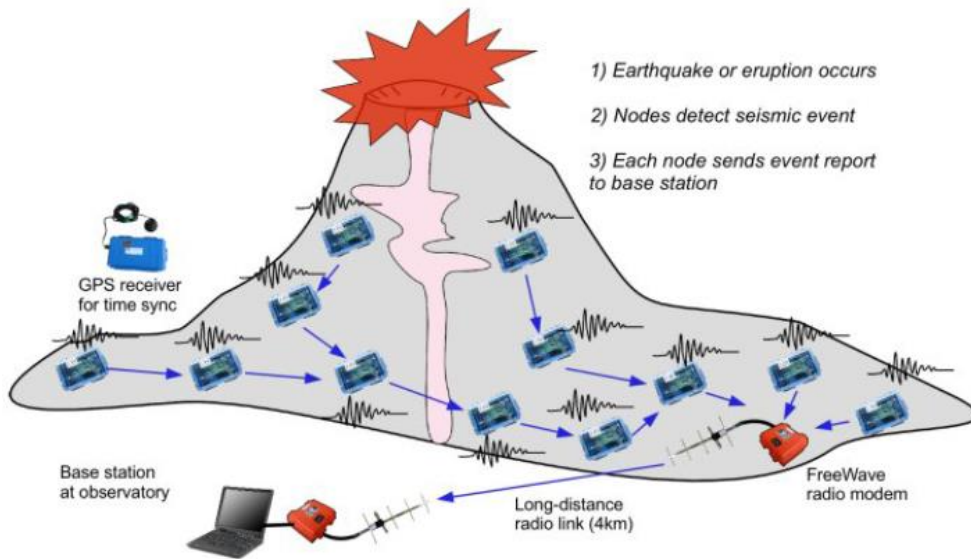
# Communication architectures



# Communication architectures

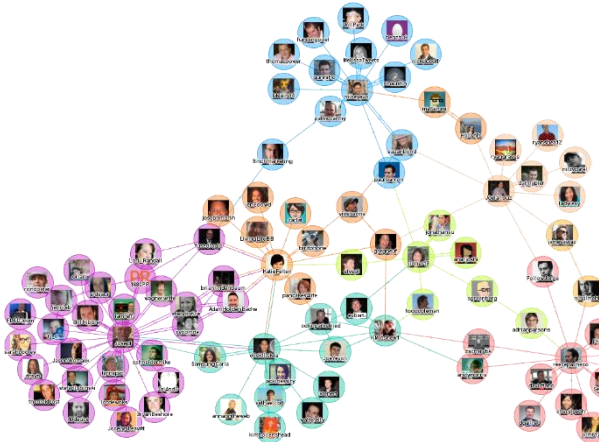


# Graph based WSN models

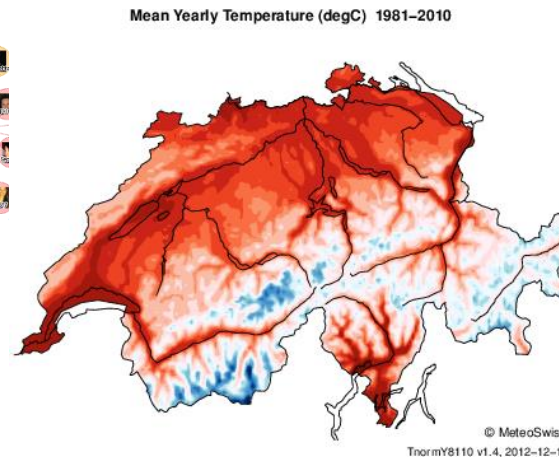
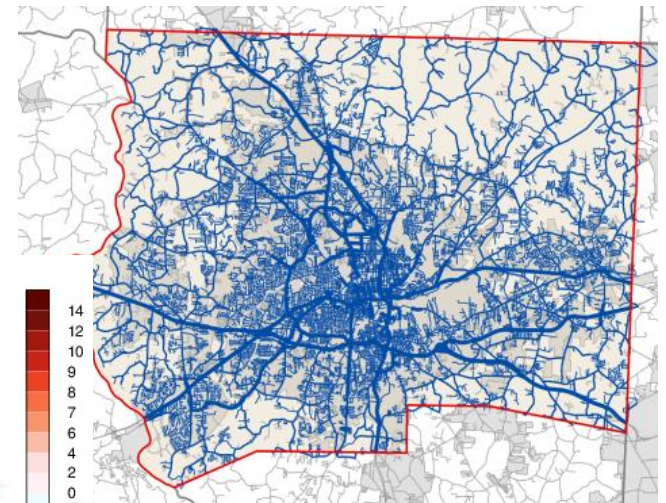


# Types of signals on graphs

## Social networks



## Electrical networks



## Environmental monitoring



# Modeling signals on graphs

Edge weight  $\leftrightarrow$  similarity between vertices.

Known

- Social media
- Sensor network

Unknown

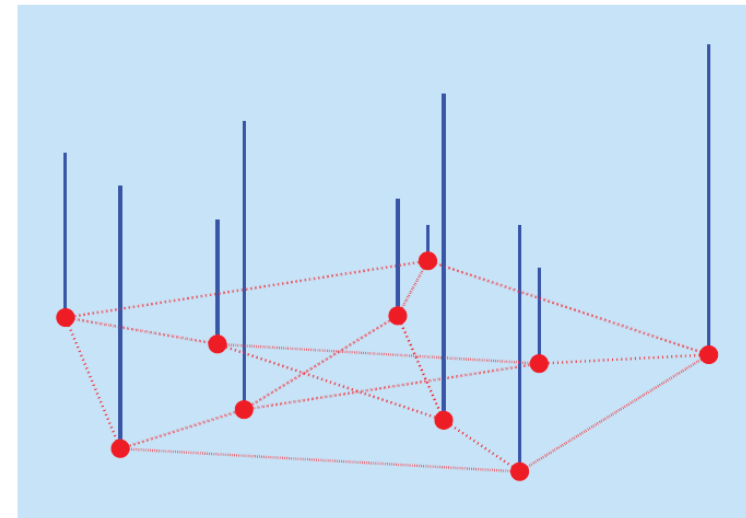
- Neuroimaging

Common data processing tasks:

- Filtering, denoising, inpainting, compression

Challenges

- What is translation, downsampling ?



The height of each blue bar represents the signal value at the vertex.



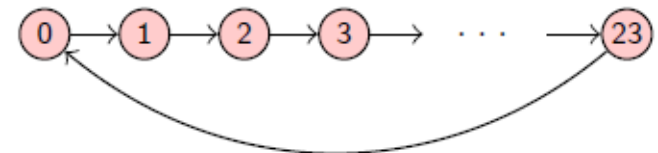
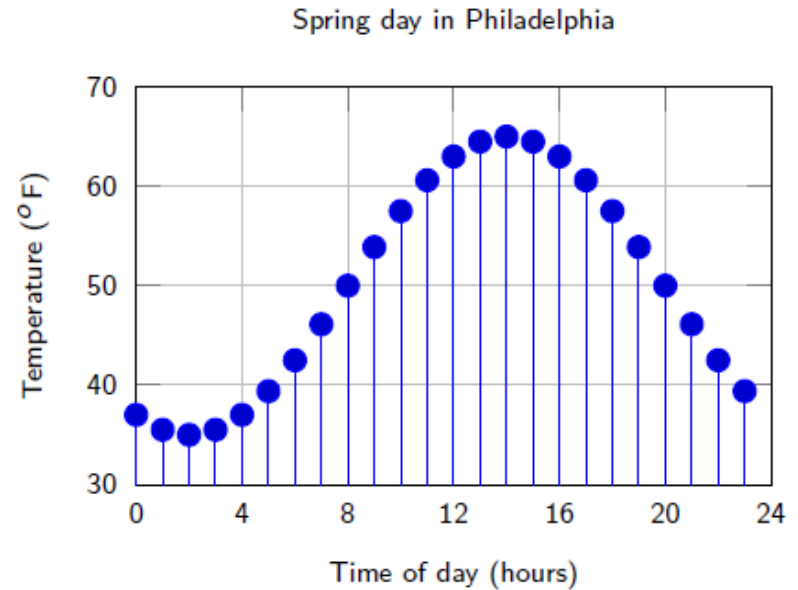
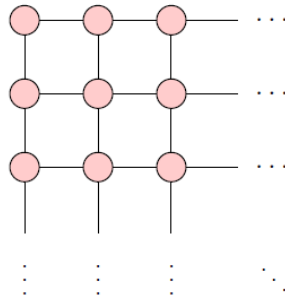
# Regular graph structures

## 1D Timeseries

- Nodes  $\leftrightarrow$  time instances
- Edges are unweighted and directed

## 2D images

- Nodes  $\leftrightarrow$  pixel
- Edges  $\leftrightarrow$  similarity

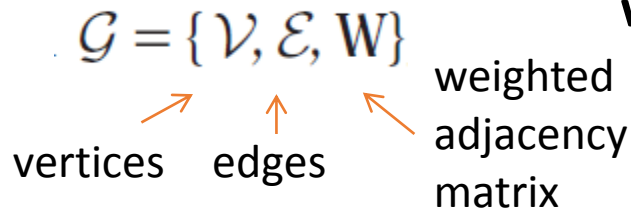




# Signals on Graphs

Graphs: generic data representation forms encoding the geometric structures of data

Applications: social networks, energy distribution networks, transportation network, **wireless sensor network**, and neuronal networks.



**weights:** distance /similarity/relationship

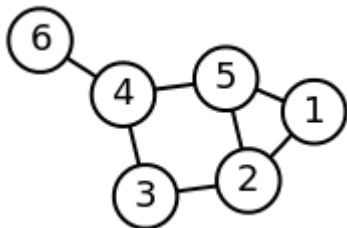
$$W_{i,j} = \begin{cases} \exp\left(-\frac{[\text{dist}(i, j)]^2}{2\theta^2}\right) & \text{if } \text{dist}(i, j) \leq \kappa \\ 0 & \text{otherwise,} \end{cases}$$

Undirected graph

Degree matrix: **D**

Adjacency matrix: **A**

Laplacian matrix: **L**



$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

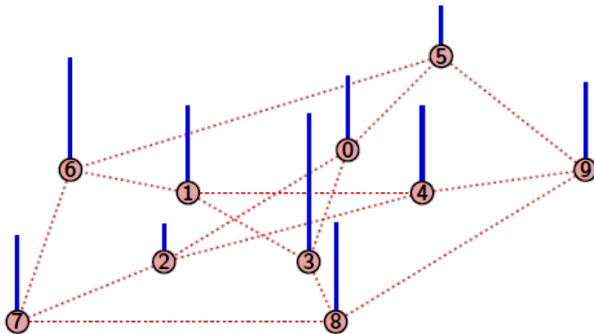
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$



# Signals on Graphs

Graph signal  $f$  in  $\mathbb{R}^N$ , where  $|V|=N$



$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_9 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.7 \\ 0.3 \\ \vdots \\ 0.7 \end{bmatrix}$$

Graph Laplacian  $\mathcal{L} := \mathbf{D} - \mathbf{W}$ ,  $\mathbf{D}$ : diagonal with sums of weights

$\mathbf{W}$ : weight matrix

Normalized Graph Laplacian  $\mathcal{L} = \mathbf{D}^{-1/2} \mathbf{L} \mathbf{D}^{-1/2}$



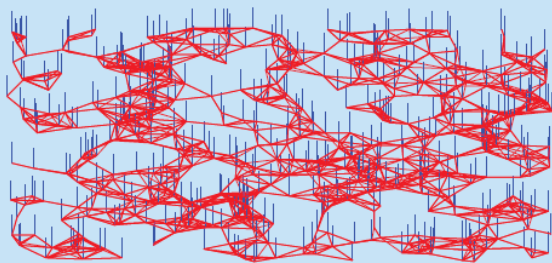
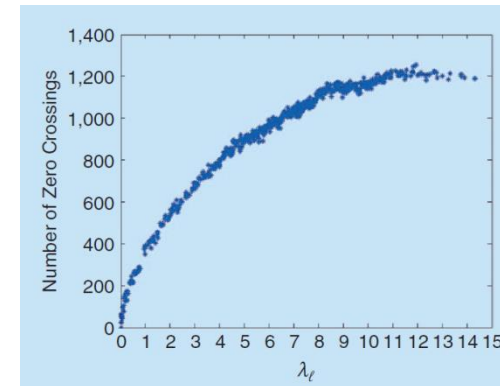
# Graph Laplacian

Spectral properties  $\mathcal{L}u_\ell = \lambda_\ell u_\ell$ ,

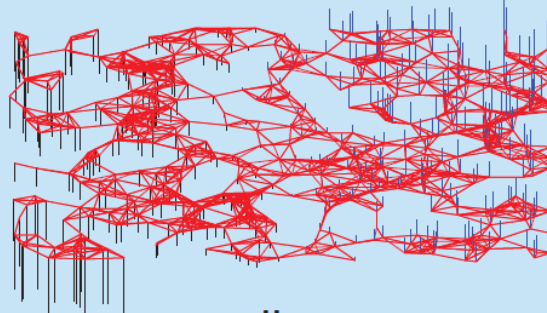
- Laplacian is Positive Semi-definite matrix
- Eigenvalues:  $0 = \lambda_1(L) \leq \lambda_2(L) \leq \dots \leq \lambda_{N-1}(L)$

$$\mathbf{x}^T \mathbf{L} \mathbf{x} = \frac{1}{2} \sum_{(i,j) \in \mathcal{E}} w_{ij} (x_i - x_j)^2 \geq 0, \text{ for all } \mathbf{x}$$

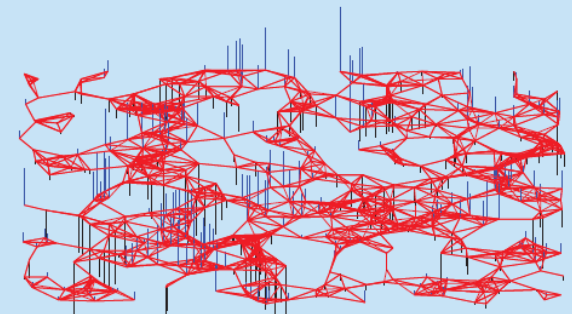
All eigenvalues are nonnegative, i.e.  $\lambda_i \geq 0$  for all  $i$



$u_0$



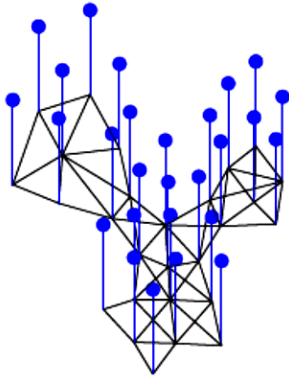
$u_1$



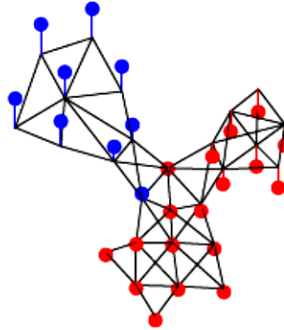
$u_{50}$

# Eigenvectors of Graph Laplacian

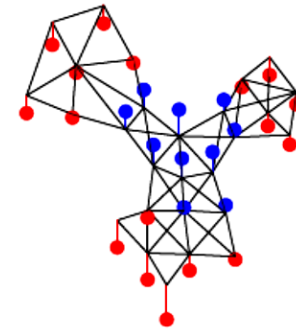
(a)  $\lambda = 0.00$



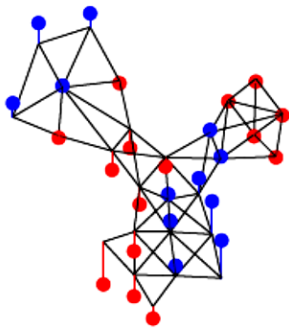
(b)  $\lambda = 0.04$



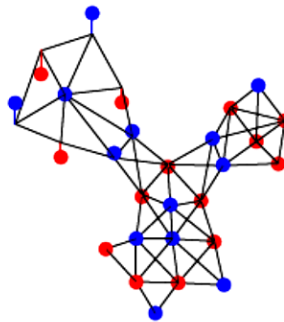
(c)  $\lambda = 0.20$



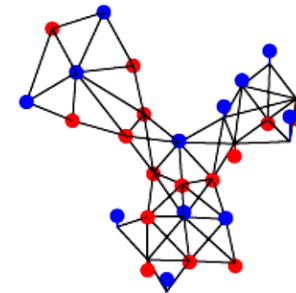
(d)  $\lambda = 0.40$



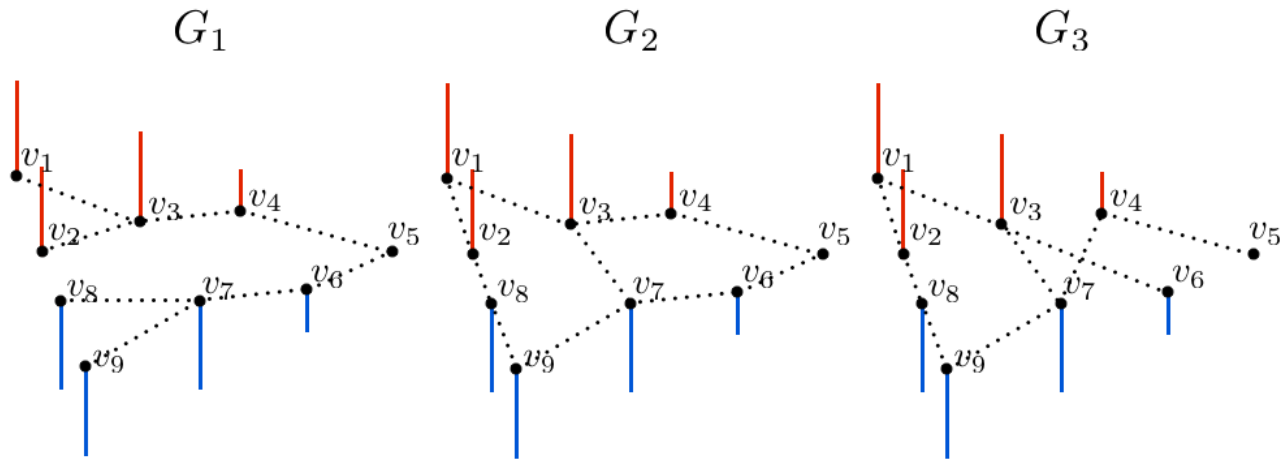
(e)  $\lambda = 1.20$



(f)  $\lambda = 1.49$



# Laplacian Regularization



- One signal  $\leftrightarrow$  many different graphs
- Only 1 leads to a smooth graph signal.
- Only  $G_1$  favors smoothness of the resulting graph signal.

# Graph Fourier Transform

Graph based approximation  $\hat{f}(\lambda_\ell) := \langle \mathbf{f}, \mathbf{u}_\ell \rangle = \sum_{i=1}^N f(i) u_\ell^*(i).$

Smoothness w.r.t. graph  $\| \mathbf{f} \|_{\mathcal{L}} := \| \mathcal{L}^{\frac{1}{2}} \mathbf{f} \|_2 = \sqrt{\mathbf{f}^T \mathcal{L} \mathbf{f}} = \sqrt{S_2(\mathbf{f})}.$

Graph spectral filtering  
(regularization)

$$\min_{\mathbf{f}} \{ \| \mathbf{f} - \mathbf{y} \|_2^2 + \gamma S_p(\mathbf{f}) \},$$

↓

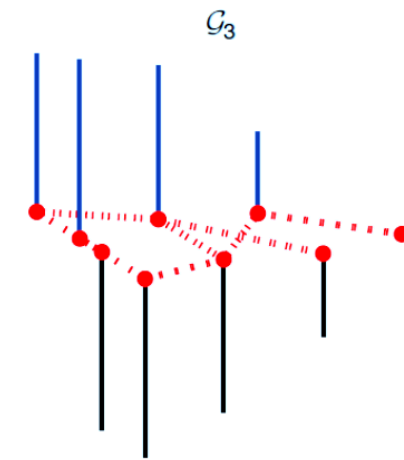
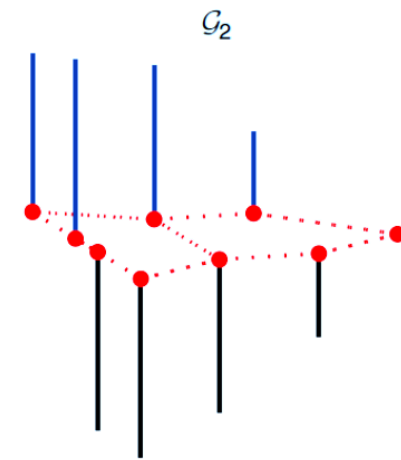
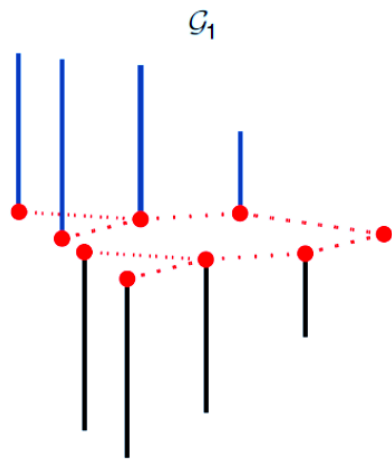
$$\operatorname{argmin}_{\mathbf{f}} \{ \| \mathbf{f} - \mathbf{y} \|_2^2 + \gamma \mathbf{f}^T \mathcal{L} \mathbf{f} \}.$$

Connectivity of the graph -> encoded in graph Laplacian

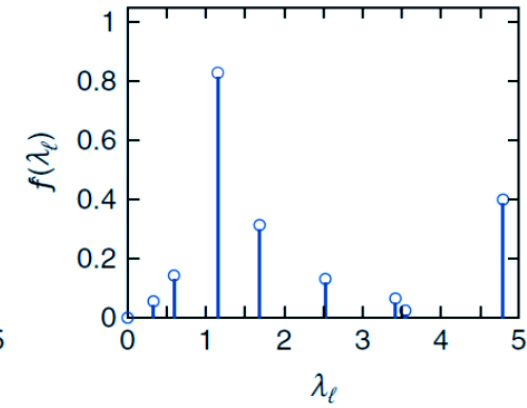
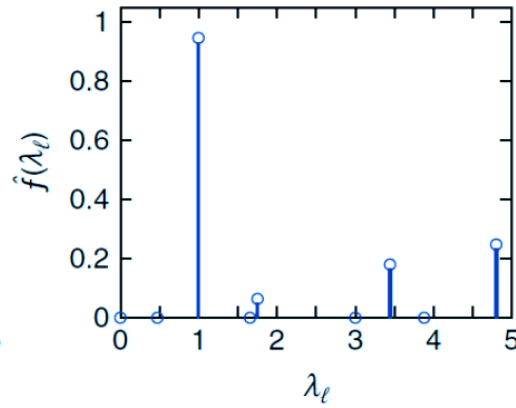
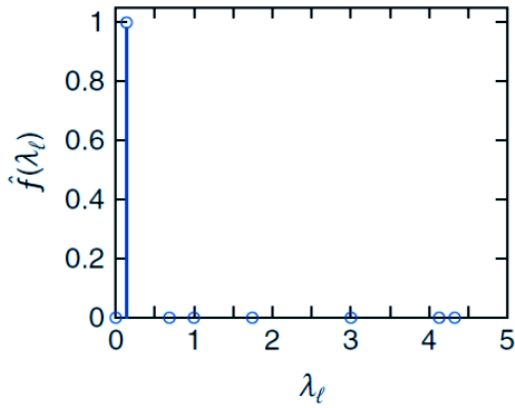
Define both a graph Fourier transform (graph Laplacian eigenvectors)

Different notions of smoothness





(a)



$$\mathbf{f}^T \mathcal{L}_1 \mathbf{f} = 0.14,$$

$$\mathbf{f}^T \mathcal{L}_2 \mathbf{f} = 1.31,$$

$$\mathbf{f}^T \mathcal{L}_3 \mathbf{f} = 1.81,$$





# Graph Fourier Transform

A graph filter is a system  $\tilde{\mathbf{s}} = \mathbf{H}(\mathbf{s})$

Equivalent  $\tilde{\mathbf{s}} = \mathbf{H}(\mathbf{s}) = h(\mathbf{A})\mathbf{s}$ .

Where  $h(\mathbf{A}) = h_0 \mathbf{I} + h_1 \mathbf{A} + \dots + h_L \mathbf{A}^L$ .

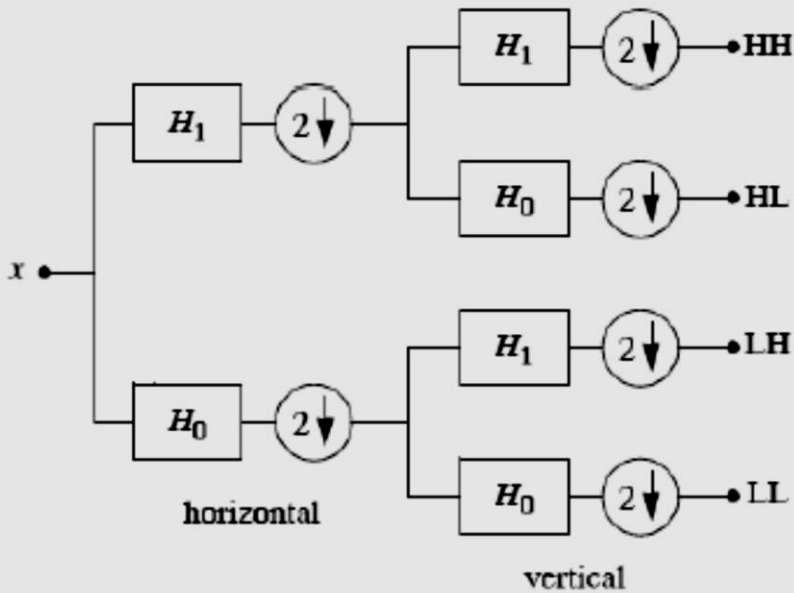
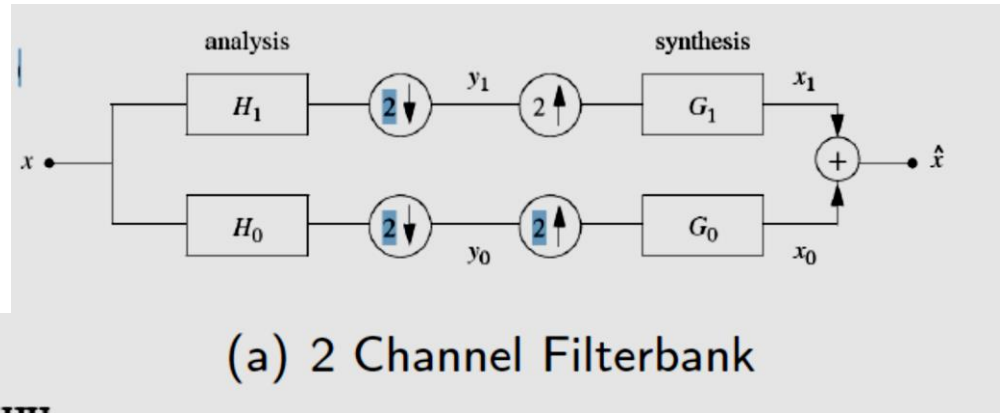
Jordan decomposition  $\mathbf{A} = \mathbf{V} \mathbf{J} \mathbf{V}^{-1}$

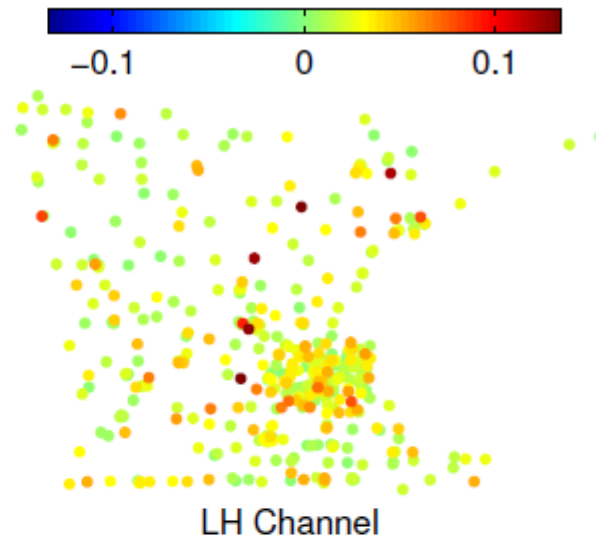
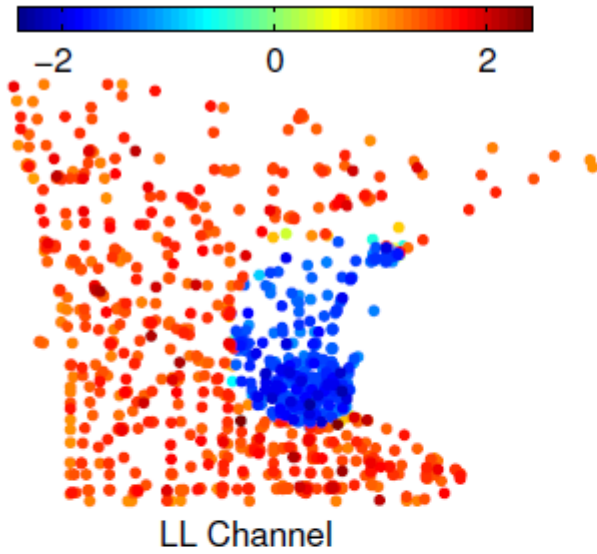
Graph Fourier Transform  $\hat{\mathbf{s}} = \mathbf{F} \mathbf{s} = \mathbf{V}^{-1} \mathbf{s}$



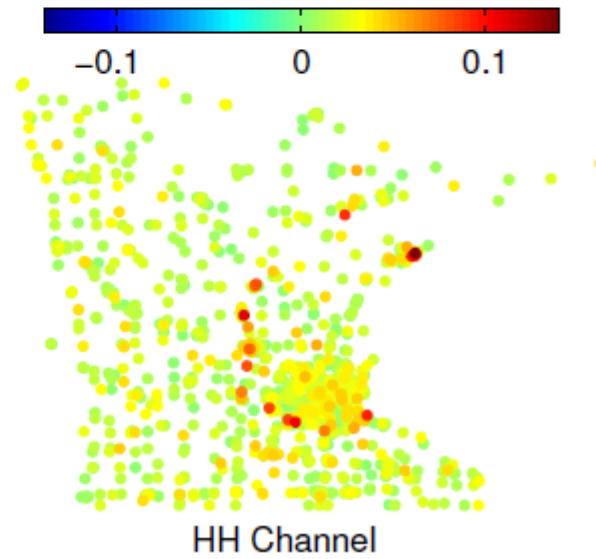
# Filters on graphs

## Wavelet filterbank





Empty Channel



HL Channel



# Spatial Signal Graphs

1-hop averaging transform

$$y[n] = \frac{1}{d_n} \sum_{m=1}^N A[n, m]x[m]$$

1-hop difference transform

$$y[n] = \frac{1}{d_n} \sum_{m=1}^N A[n, m](x[n] - x[m])$$

$$\mathbf{y} = \mathbf{D}^{-1} \mathbf{A} \mathbf{x} = \mathbf{P}_{rw} \mathbf{x}$$

$$\mathbf{y} = \mathcal{L}_{rw} \mathbf{x} = \mathbf{x} - \mathbf{P}_{rw} \mathbf{x}$$



# Spectral anomaly detection in WSN

Decomposition of Laplacian  $\mathcal{L}_G = \mathbf{U}_G \mathbf{\Lambda}_G \mathbf{U}_G^t$

Alternative approach  $h(\mathcal{L}_G) = \mathbf{U}_G (h(\mathbf{\Lambda}_G)) \mathbf{U}_G^t$

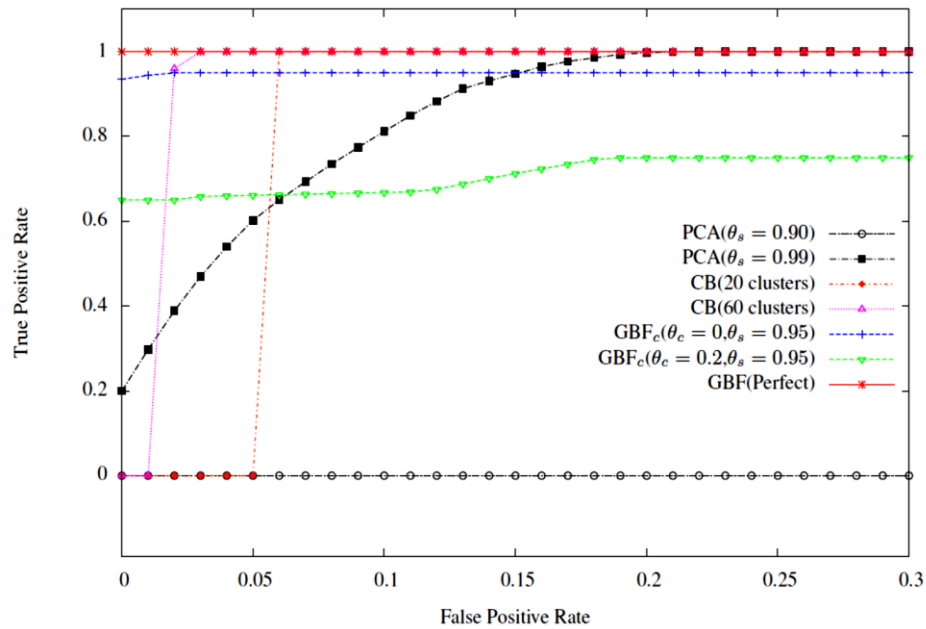
Graph construction  $[w_{i,j}]_b = \exp\left(-\frac{(1 - \|\rho(i,j)\|_1)^2}{\Delta_c^2}\right) \cdot \exp\left(-\frac{\tilde{D}(i,j)^2}{\Delta_d^2}\right)$

Data fit in graph  $\sigma_l^2 = s^2[\mathbf{u}_l^t \mathbf{X}]$  where  $s^2[\mathbf{p}^t]$  is the sample variance

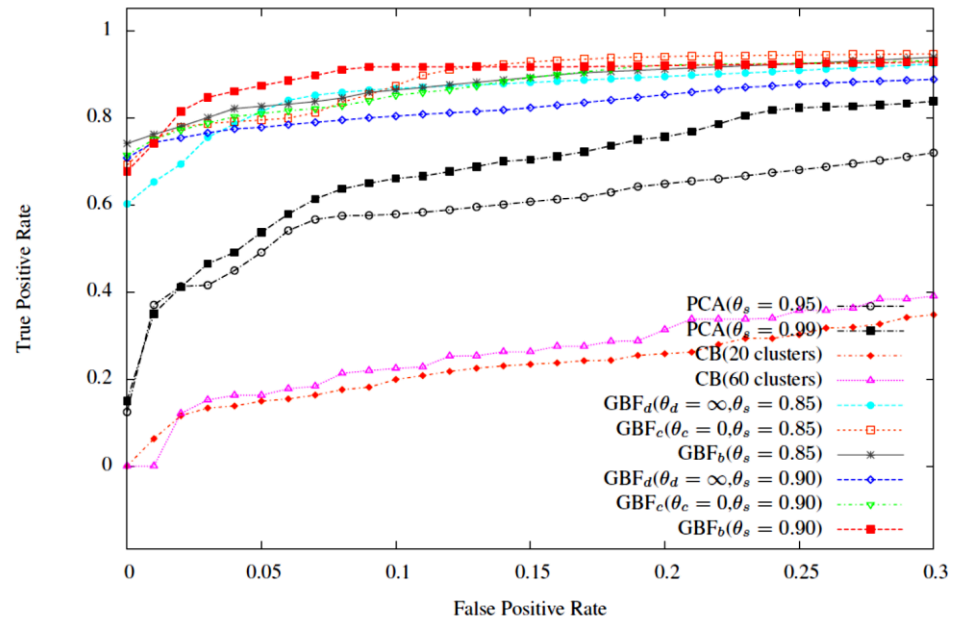
Target ratio  $\arg \max_c \sum_{l=1}^c \frac{\sigma_l^2}{\sigma_T^2}$  subject to  $\sum_{l=1}^c \frac{\sigma_l^2}{\sigma_T^2} \leq \theta_s$



# Global



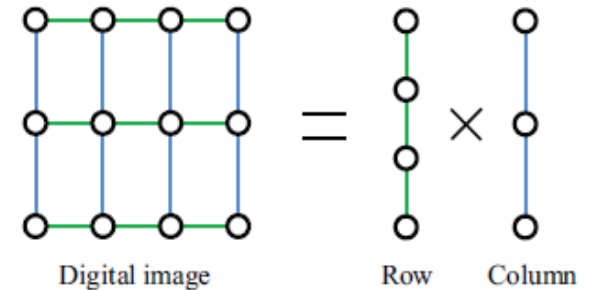
# Distributed



# Product Graphs

- Assume:  $G_1=(V_1,A_1)$  and  $G_2(V_2,A_2)$

- Product graph:  $G = G_1 \diamond G_2 = (V, A_\diamond)$ ,

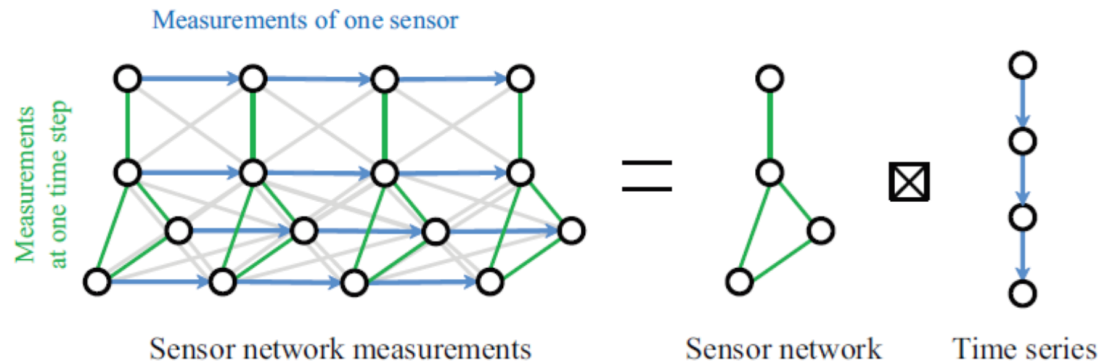


- Kronecker:

$$\mathbf{A}_\otimes = \mathbf{A}_1 \otimes \mathbf{A}_2.$$

- Cartesian:

$$\mathbf{A}_\times = \mathbf{A}_1 \otimes \mathbf{I}_{N_2} + \mathbf{I}_{N_1} \otimes \mathbf{A}_2.$$

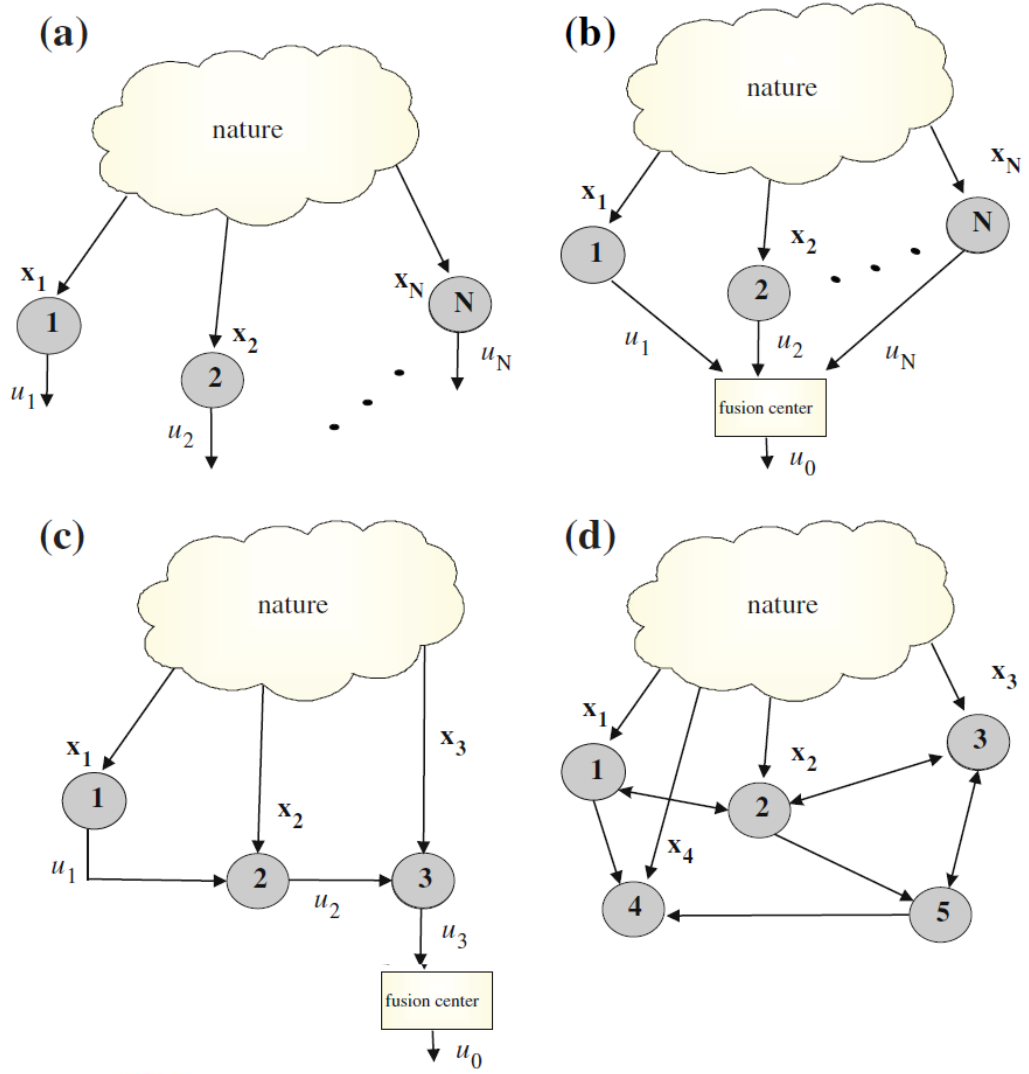


- Strong:

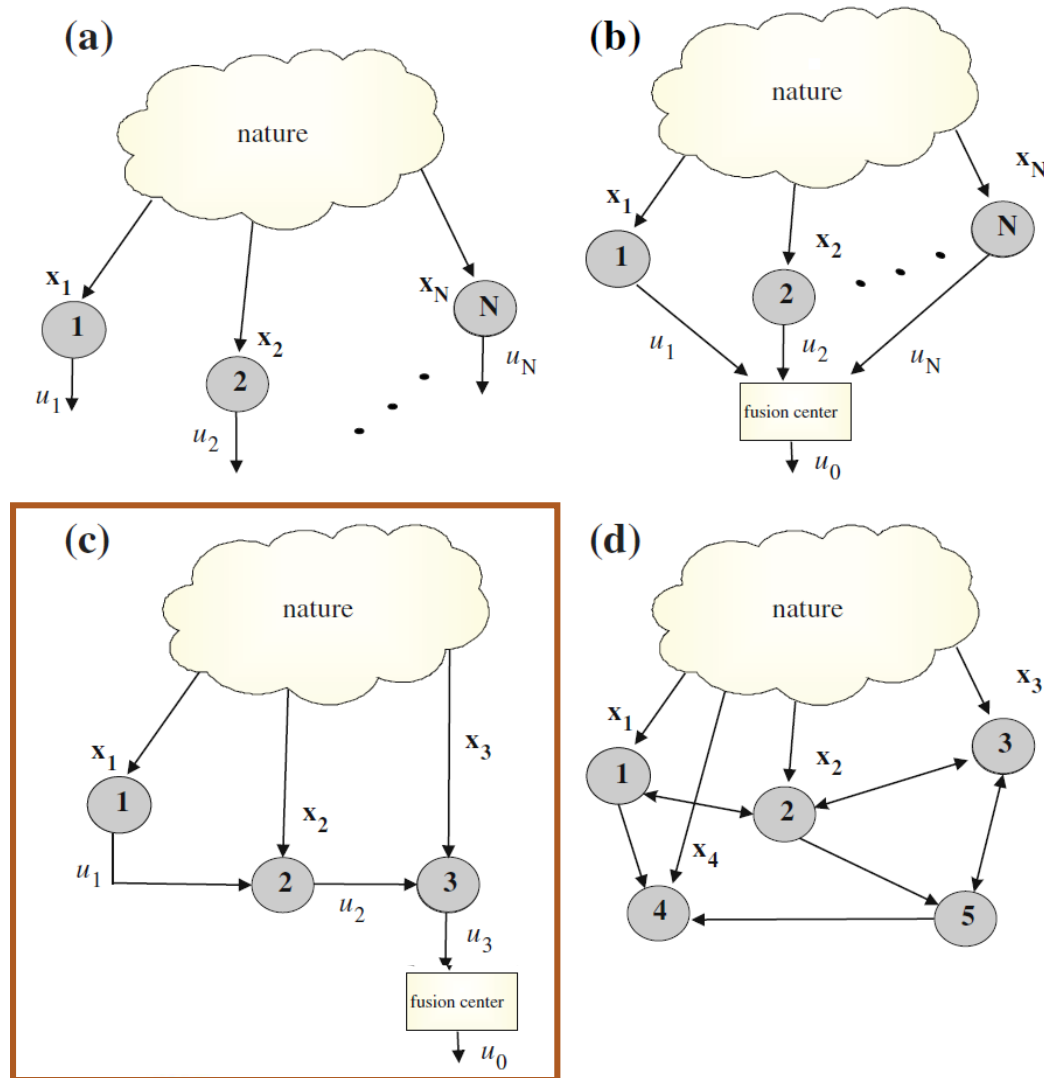
$$\mathbf{A}_\boxtimes = \mathbf{A}_1 \otimes \mathbf{A}_2 + \mathbf{A}_1 \otimes \mathbf{I}_{N_2} + \mathbf{I}_{N_1} \otimes \mathbf{A}_2.$$



# Communication architectures



# Communication architectures

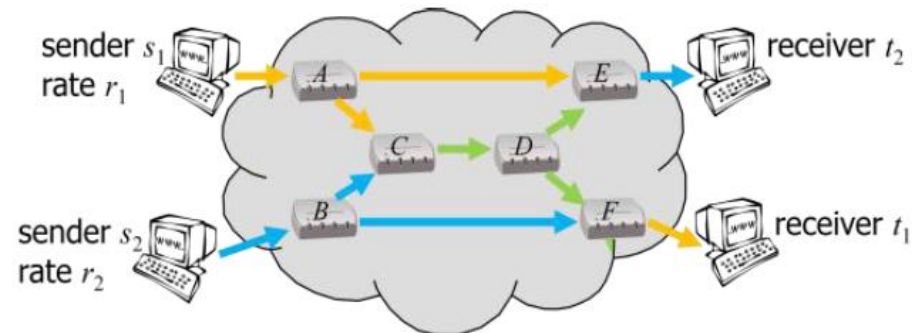
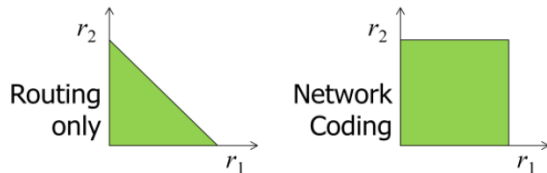


# Network Coding (NC)

Typical routing: Each message on an output link must be a copy of a message that arrived earlier on an input link

Network coding: each message sent on a node's output link can be some function or "mixture" of messages that arrived earlier on the node's input links

Motivation: **improve throughput**



Minimize

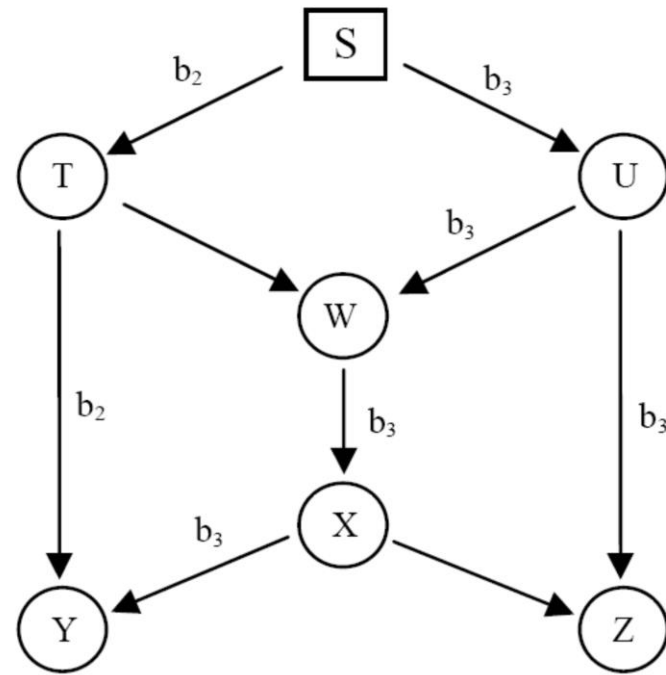
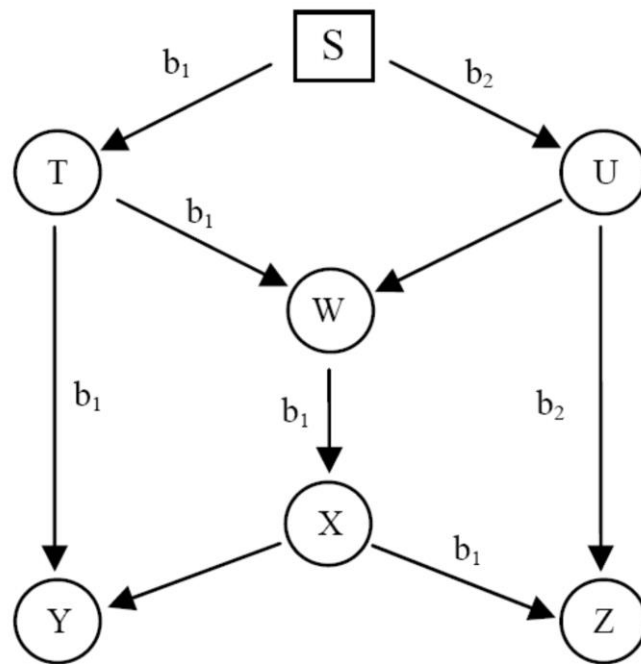
- Energy per bit
- Delay

R. Ahlswede, N. Cai, S.-Y. R. Li, and R.W. Yeung, "Network information flow,"  
*IEEE Trans. on Information Theory*, vol. 46, no. 4, July 2000

# Typical Unicast

Without network coding

- Simple store and forward
- Multicast rate of 1.5 bits per time unit



# Traditional Method

A B C



# Network Coding

A B C



# Unicast with NC

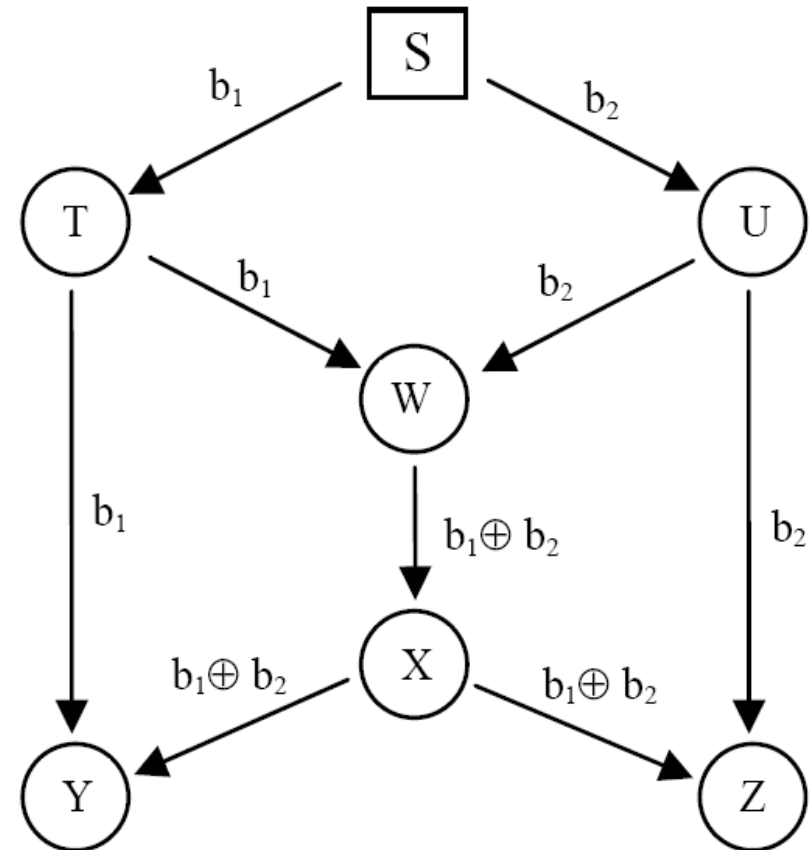
## With network coding

X-OR  $\rightarrow$  one of the simplest form of coding

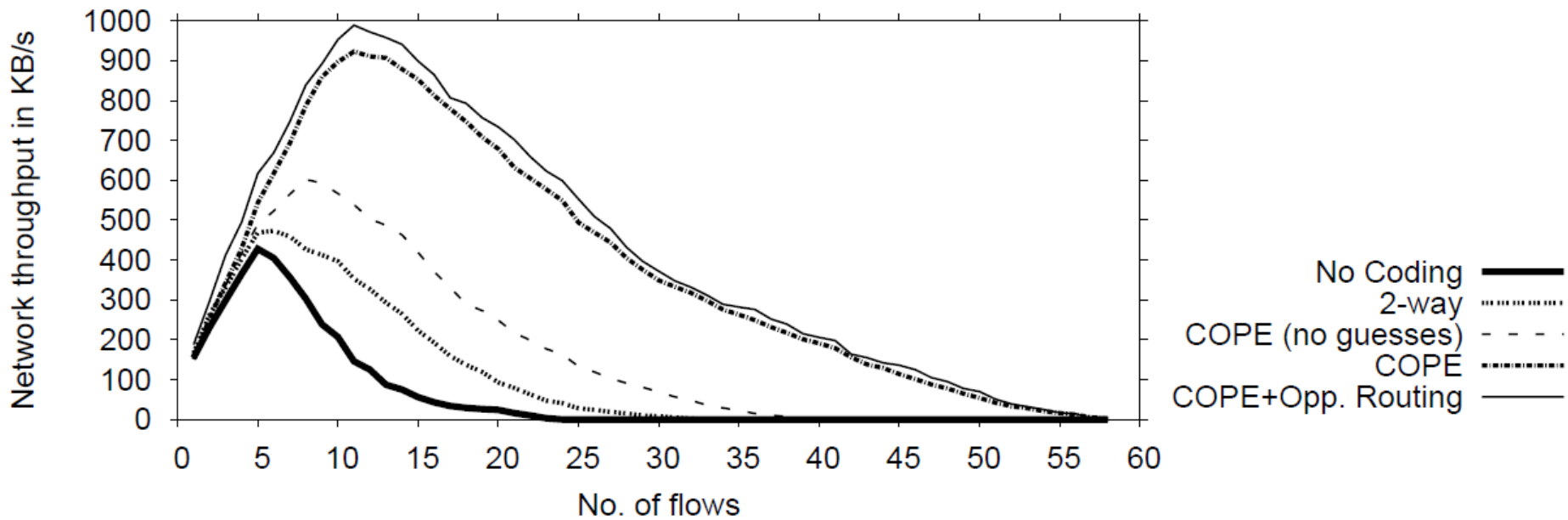
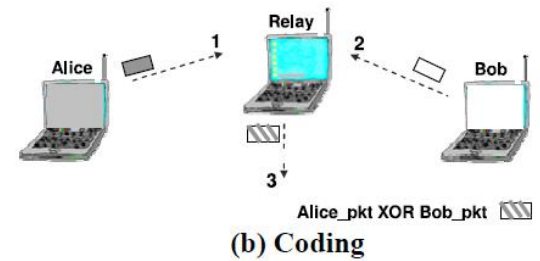
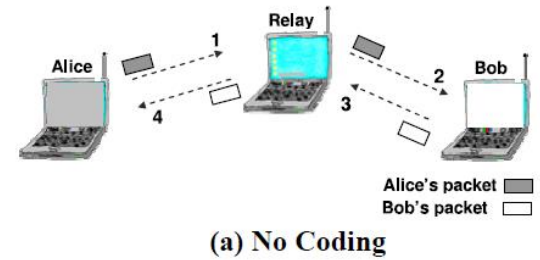
Multicast rate of 2 bits per time unit

## Disadvantages:

- Coding/decoding scheme has to be agreed upon beforehand



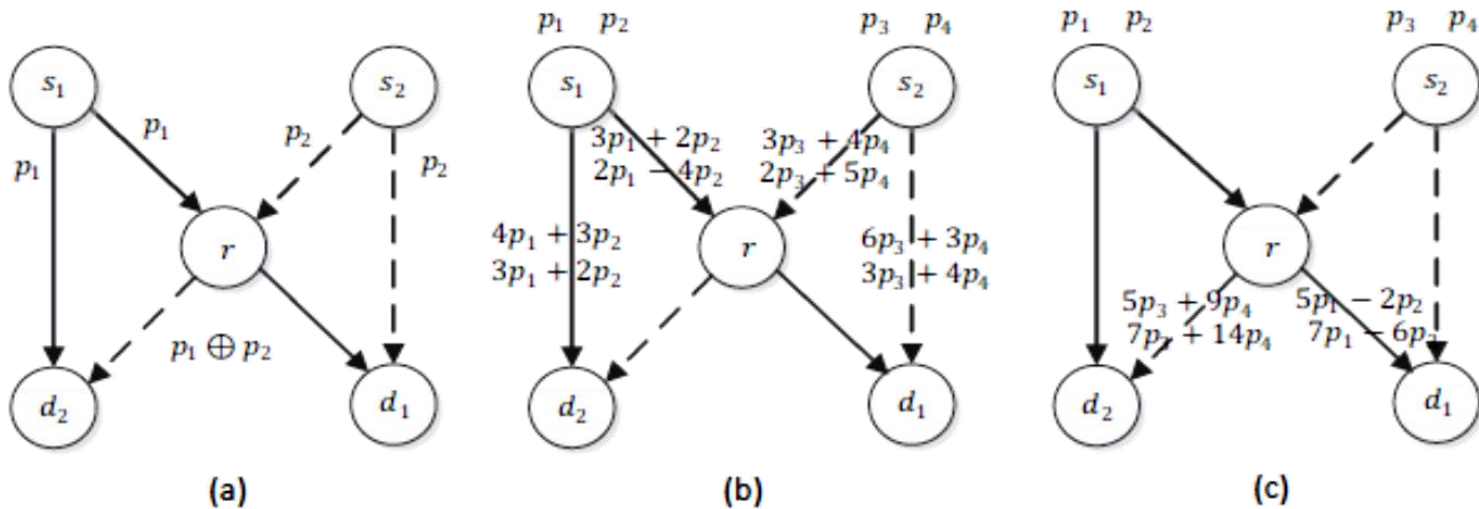
# NC performance





# Generalize to packets

- Operate on packets instead of on bit-streams



# NC encoding & decoding

Message

$$y(e) = \sum_{e': \text{out}(e')=v} m_e(e')y(e')$$

$$y(e) = \sum_{i=1}^h g_i(e)x_i$$

Encoding vector

$$\overrightarrow{m}(e) = [m_e(e')]_{e': \text{out}(e')=v}$$

$$\overrightarrow{g}(e) = [g_1(e), \dots, g_h(e)]$$

Decoding

$$\begin{bmatrix} y(e_1) \\ \vdots \\ y(e_h) \end{bmatrix} = \begin{bmatrix} g_1(e_1) & \cdots & g_h(e_1) \\ \vdots & \ddots & \vdots \\ g_1(e_h) & \cdots & g_h(e_h) \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_h \end{bmatrix} = G_t \begin{bmatrix} x_1 \\ \vdots \\ x_h \end{bmatrix}$$



# NC encoding & decoding

**Node t can recover the source symbols  $x_1, \dots, x_h$  as long as the matrix  $G_t$ , formed by the global encoding vectors, has (full) rank.**

$$\begin{bmatrix} x_1 \\ \vdots \\ x_h \end{bmatrix} = G_t^{-1} \begin{bmatrix} y(e_1) \\ \vdots \\ y(e_h) \end{bmatrix}$$

$G_t$  will be invertible w.h.p. if local encoding vectors are random and the field size is sufficiently large

R. Koetter, M. Medard, "An algebraic approach to network coding", *IEEE/ACM Trans. on Networking*, 2003



# Practical NC: Random NC

## Issues

- Synchronous / asynchronous packet delivery
- Varying capacity edges
- Packet delays and drops
- Central coding pattern knowledge

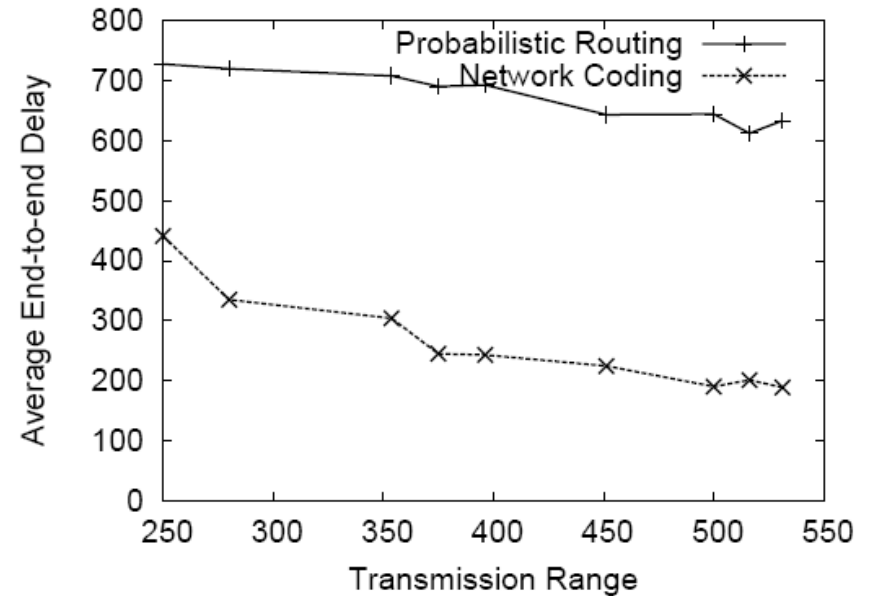
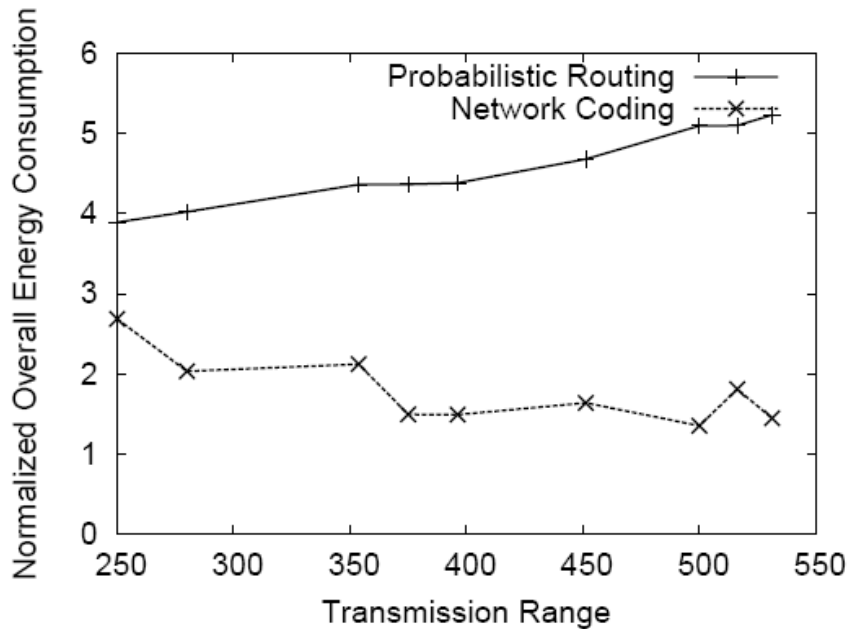
Random NC: random linear coefficients in a finite field and send the encoding vector within the same packet

**Packetization:** Header removes need for centralized knowledge of graph topology and encoding/decoding functions

**Buffering:** Nodes store within their buffers the received packets. Allows asynchronous packets arrivals & departures with arbitrarily varying rates, delay, loss



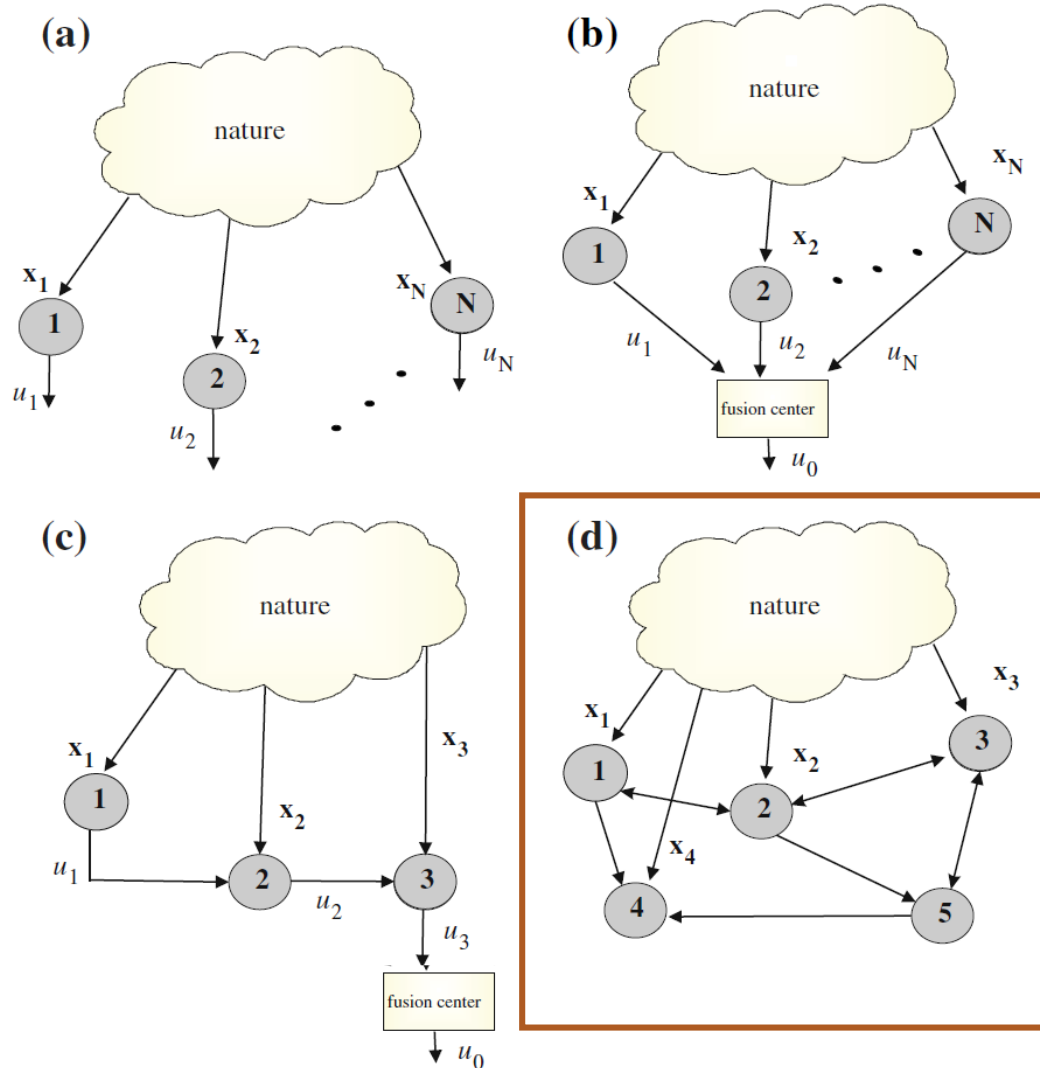
# Random NC simulation results



**Energy consumption:** number of transmissions and receptions needed to gather all the required packets

**Delay:** number of time units needed to decode all the required packets

# Communication architectures



# In-network processing

## Collaborative signal processing:

- Exploit local computational resources -> reduce data transmissions

Power(Communications) > Power(Processing)

- Applications

- Detection, Classification, Parameter Estimation, Tracking...

- Assumptions

- Specialized routing protocols
- spatio-temporal smoothness

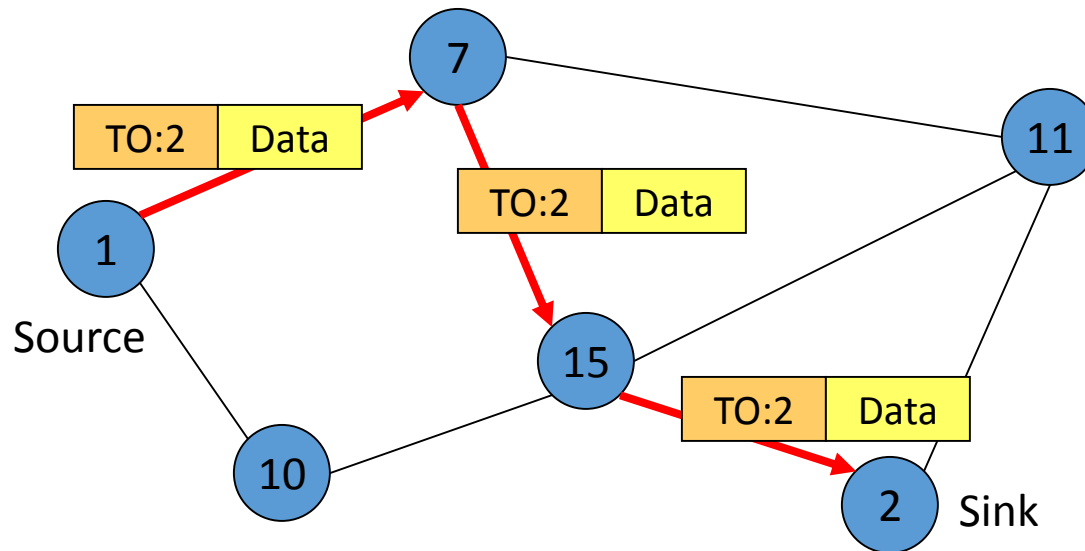




# Address-based routing vs. data-centric forwarding

- Address-based routing

- Directed towards a well-specified *particular destination* (sink)
- Support for unicast, multicast, and broadcast messages



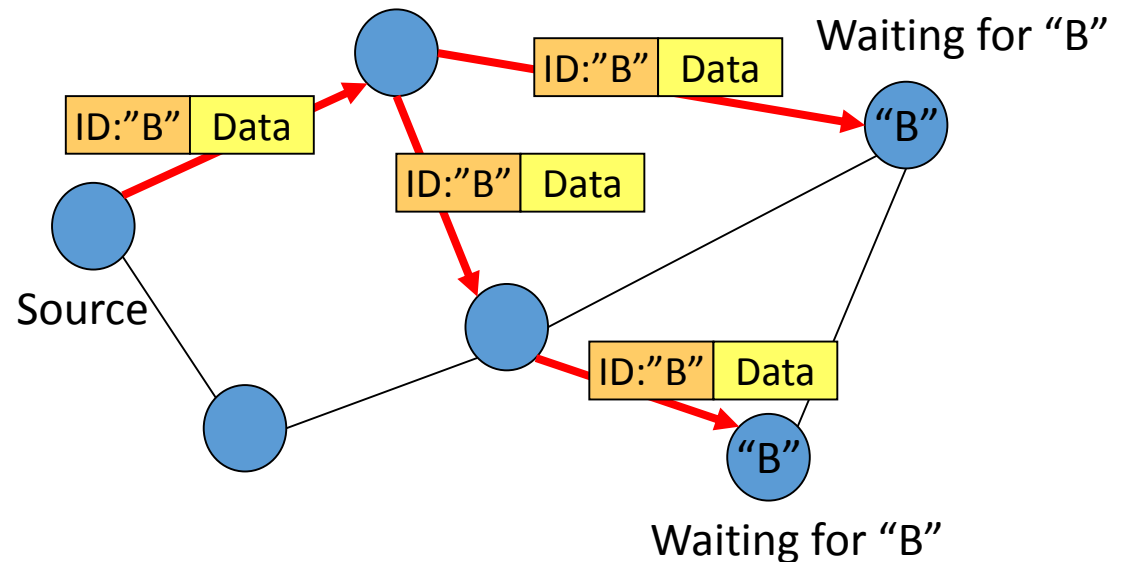
# Data-Centric Networking

The traditional communication paradigm focuses on the relationship between communicating peers

In WSNs, the application is not interested in the *identity* of the nodes, but rather in the information about the *physical environment*

## Objectives

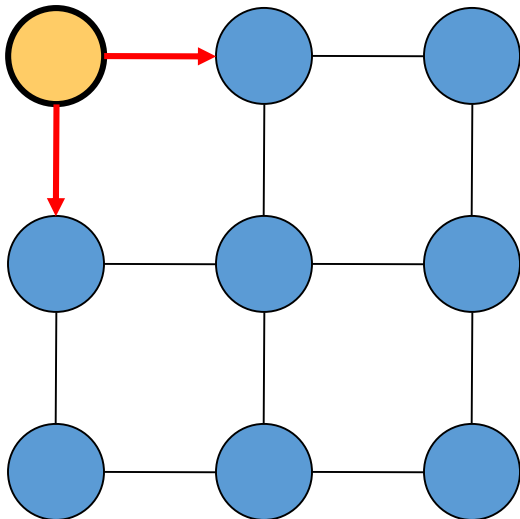
- In-network aggregation
- Data-centric addressing
- Decoupling in time
- Fault-tolerance
- Scalability



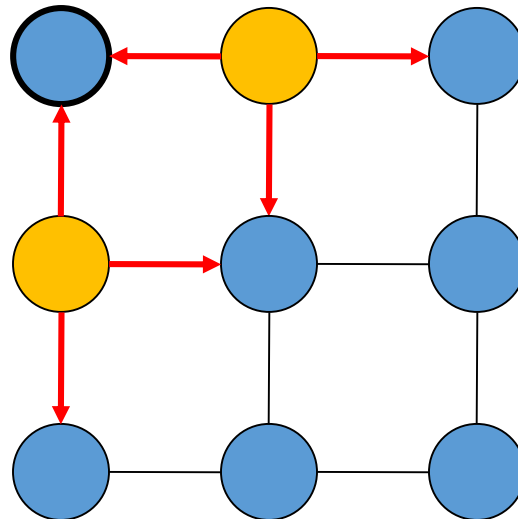
# Flooding

- Basic mechanism:
  - Each node that receives a packet re-broadcasts it to all neighbors
  - The data packet is discarded when the maximum hop count is reached

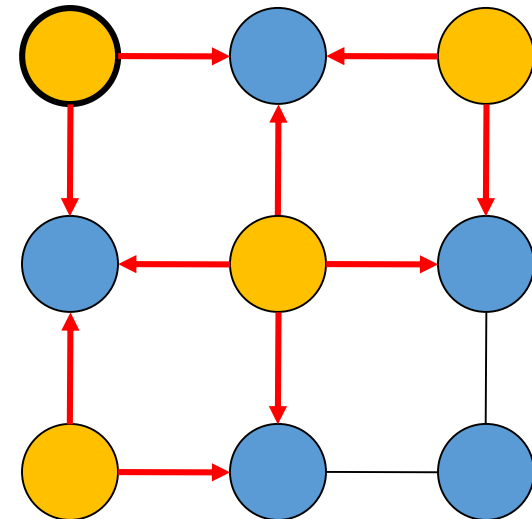
Step 1



Step 2



Step 3



# Flooding

Simplest method for message delivery from observation node to sink

- NO routing table NOR next hop estimation
- On receiving the packet, a sensor just rebroadcasts it

(+) Low computing complexity

(+) No memory for path caching

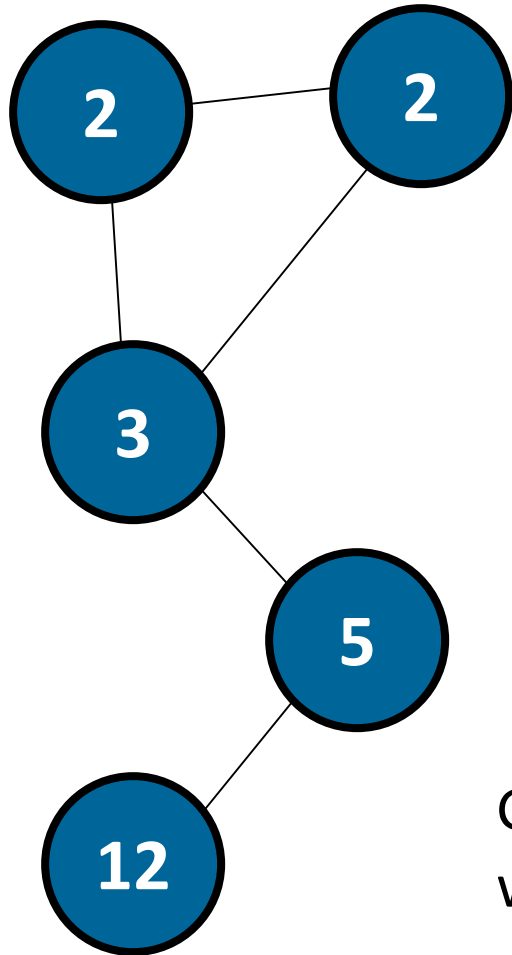
(-) Implosion: duplicated messages are received

(-) *Overlay: flooding of overlapping data*

(-) Resource blindness



# Distributed Aggregation



- Every node has a measurement (e.g sensing temperature)
- Every node wants to access the **global average**
- Want a truly distributed, localized and robust algorithm to compute the average.

Goal: every node gets  $(2+2+3+5+12)/5=4.8$  with the **minimum** energy cost



# Consensus algorithms

Having a set of agents to agree upon a certain value (usually global function) using only local information exchange (local interaction)

## Objectives:

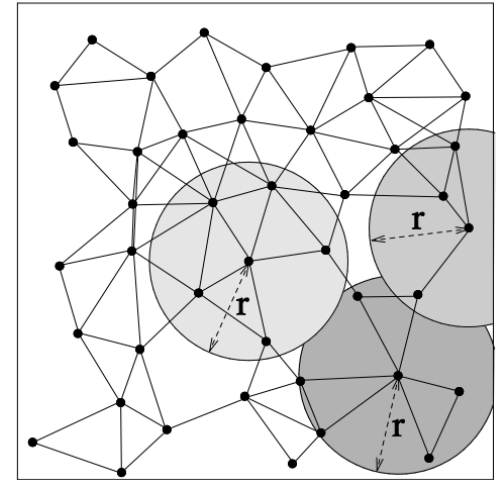
- Distributed computation of general functions
- Computational efficient
- Robust to failures
- Independent of topology

## **Distributed Average Consensus**

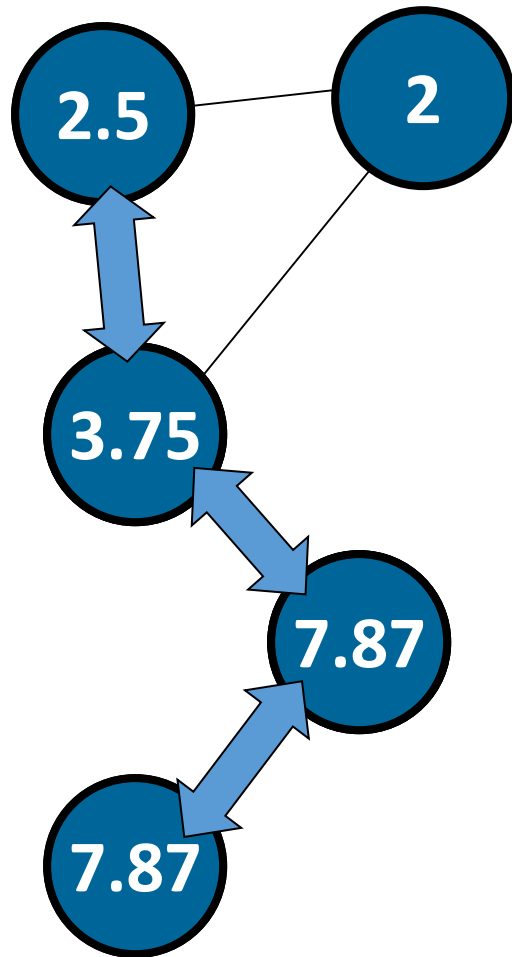
Nodes measure  $\rightarrow$  average  $x_i \rightarrow x_{ave} = \sum x_i/n$

## Assumptions

- Nodes their neighbors (location)
- Dynamic network topology



# Gossip Algorithms for Aggregations



- Start with initial measurement as an estimate for the average and update
- Each node interacts with a random neighbor and both compute **pairwise** average (one **update**)
- Converges to true average
- Useful building block for more complex problems

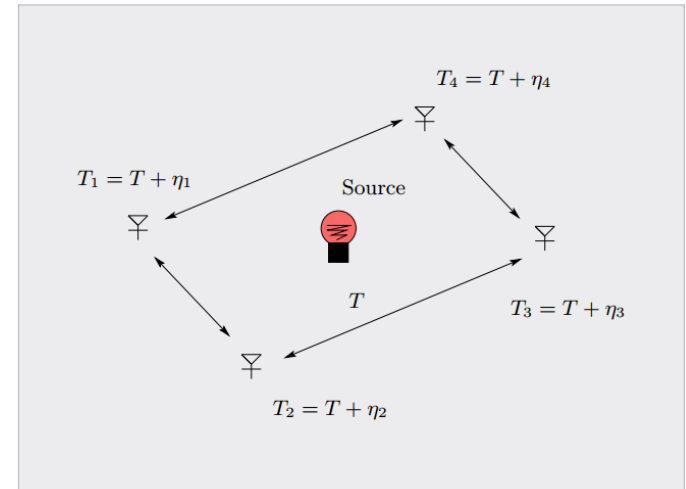
# Gossip Algorithms

One solution to distributed consensus

- Each node ( $n$  nodes in total) holds an estimate
- Goal: for every node estimate  
average of all  $n$  initial values

## Iterative + random

- At each iteration:
  - random groups communicate  
& average
- Local estimation  $\rightarrow$  global consensus
- Q: Time? Packets? Quality? Synchronization?





# What is a Random Walk

- Given a graph and a starting point (node), we select a neighbor of it at random, and move to this neighbor;
- Then we select a neighbor of this node and move to it, and so on;
- The (random) sequence of nodes selected this way is a **random walk** on the graph



# What is a random walk

- $n \times n$  **Adjacency matrix A.**

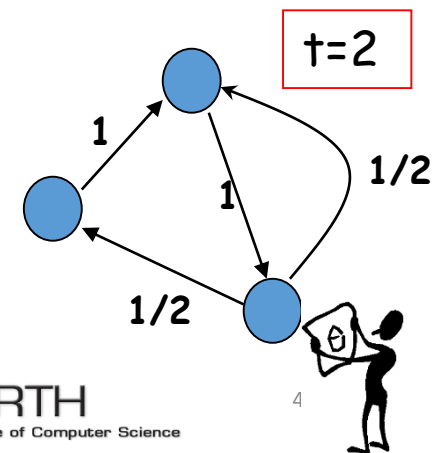
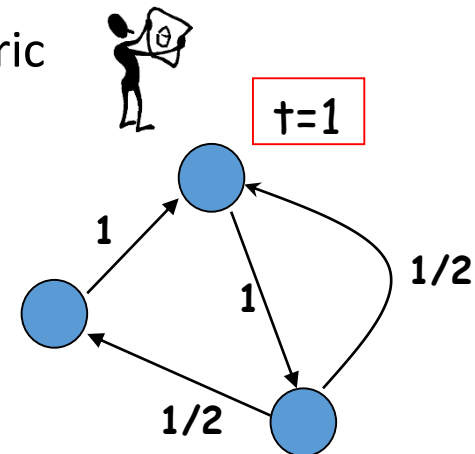
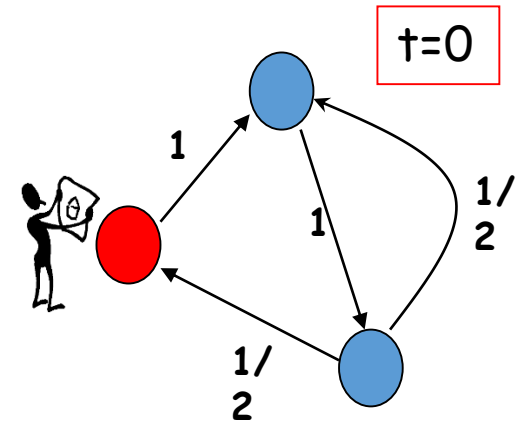
- $A(i,j)$  = weight on edge from  $i$  to  $j$
- If the graph is undirected  $A(i,j)=A(j,i)$ , i.e.  $A$  is symmetric

- $n \times n$  **Transition matrix P.**

- $P$  is row stochastic (doubly for undirected)
- $P(i,j)$  = probability of stepping on node  $j$  from node  $i$   
 $= A(i,j)/\sum_i A(i,j)$

- $n \times n$  **Laplacian Matrix L.**

- $L(i,j)=\sum_i A(i,j)-A(i,j)$
- Symmetric positive semi-definite for undirected graphs
- Singular



# Probability Distributions

- $x_t(i)$  = prob. that the surfer is at node  $i$  at time  $t$
- $x_{t+1}(i) = \sum_j (\text{Prob. of being at node } j) * \text{Pr}(j \rightarrow i) = \sum_j x_t(j) * P(j,i)$
- $x_{t+1} = x_t P = x_{t-1} * P * P = x_{t-2} * P * P * P = \dots = x_0 P^t$

When one keeps walking for a long time?

- For the stationary distribution  $v_0$  we have  $v_0 = v_0 * \pi$

For connected, non-bipartite graphs

$$\pi(v) = \text{node degree}/2 * \#edges = d(v)/2m$$

The more neighbors you have, the more chance you'll be reached



# Cover time in Graphs

Given a graph  $G$ , let  $T_{\text{cover}}(u)$  be the expected length of a simple random walk that starts at node  $u$  and visits every node in  $G$  at least once.

*Cover time* of  $G \Rightarrow T_{\text{cover}}(G) = \max_{u \in G} T_{\text{cover}}(u)$ .

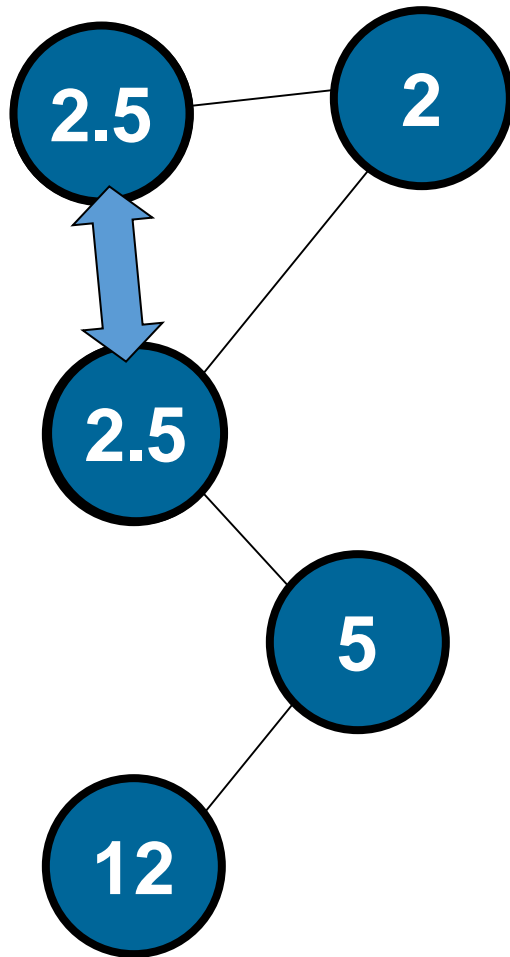
Given a random geometric graph  $G$  with  $n$  nodes, if it is a connected graph with high probability, then

$$T_{\text{ave}}(\varepsilon, n) = \Theta(n(\log n + T_{\text{mix}}(\varepsilon)))$$

A random walk visits each node once by requiring that it makes  **$C n \log n$**  steps for some  $C > 0$ .



# Standard gossip



$W(1)$					$x(0)$	$x(1)$	
$1/2$	$0$	$1/2$	$0$	$0$	$2$	$\leftarrow$ $\leftarrow$ =	$2.5$
$0$	$1$	$0$	$0$	$2$	$2$		$2$
$1/2$	$0$	$1/2$	$0$	$0$	$3$		$2.5$
$0$	$0$	$0$	$1$	$0$	$5$		$5$
$0$	$0$	$0$	$0$	$1$	$12$		$12$

$$x(t) = \mathbf{W}(t) x(t-1) = \prod_t \mathbf{W}(t) x(0)$$

$\mathbf{W}(t)$  iid random matrices



# How many messages

- $\varepsilon$ -averaging time: First time where  $x(t)$  is  $\varepsilon$ -close to the normalized true average with probability greater than  $1-\varepsilon$ .

$$T_{ave}(n, \varepsilon) = \sup_{x(0)} \inf \left\{ t : P\left(\frac{\|x(t) - x_{ave} \mathbf{1}\|}{\|x(0)\|} \geq \varepsilon\right) \leq \varepsilon \right\}$$

- $x(t) = \mathbf{W}(t) x(t-1) = \prod_t \mathbf{W}(t) x(0)$ .
- Define  $\mathbf{W} = E \mathbf{W}(t)$
- Theorem:  $\varepsilon$ -averaging time can be bounded using the spectral gap of  $\mathbf{W}$ :

$$T_{ave}[n, \varepsilon] \leq \frac{3 \log(\varepsilon^{-1})}{1 - \lambda_2(W)}$$

(Boyd, Gosh, Prabhakar and Shah, IEEE Trans. On Information Theory, June 2006)



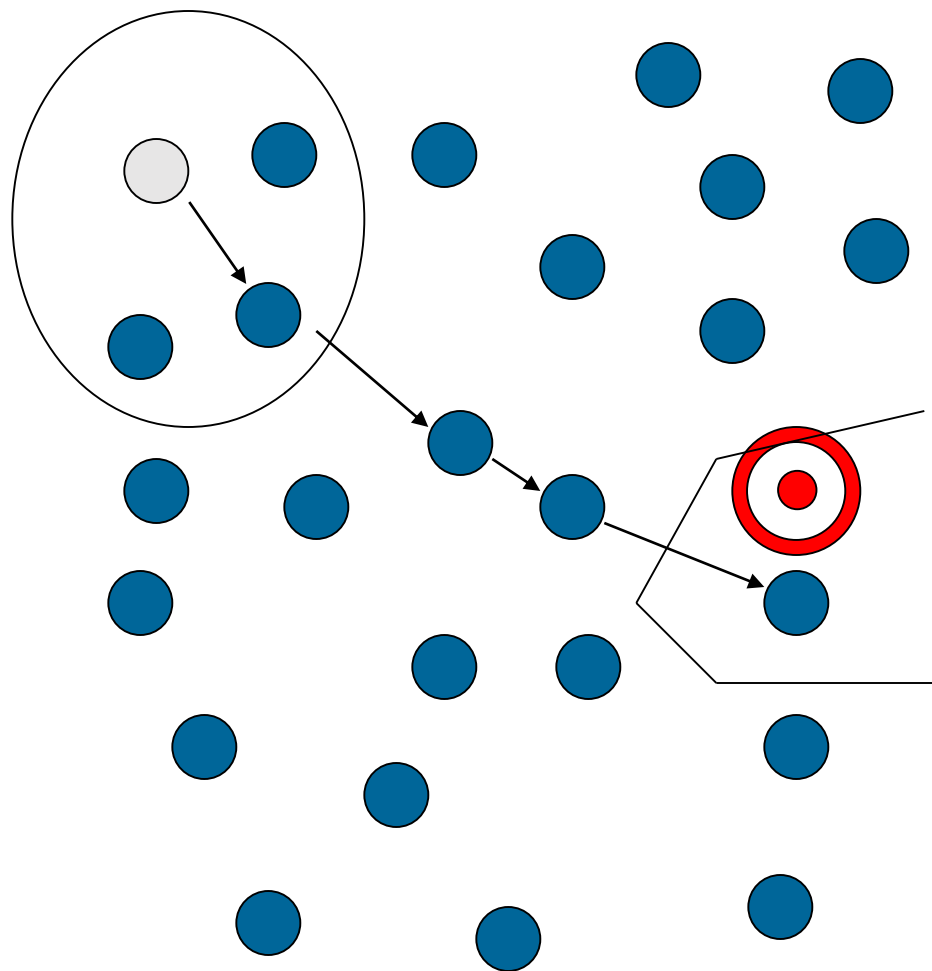
# Cost of standard Gossip

- Standard Gossip algorithms **require a lot of energy.**  
(For realistic sensor network topologies)
- **Why:** useful information performs random walks, diffuses slowly
- Can we save energy with extra information?
- **Idea:** gossip in random directions, diffuse faster.
- Assume each node knows its location and locations of 1-hop neighbors.



# Random Target Routing

- Node picks a random location (=“target”)
- Greedy routing towards the target
- Probability to receive  $\sim$  Voronoi cell area





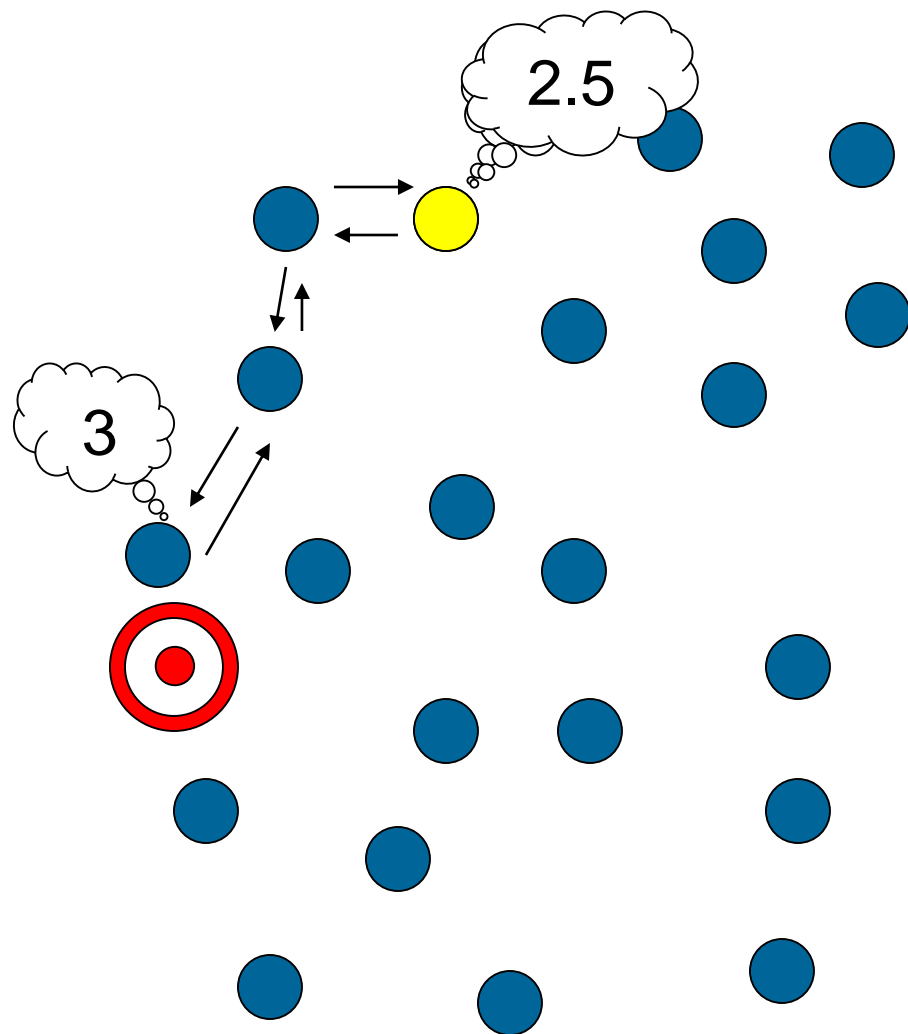
# Geographic Gossip

- Nodes use random routing to gossip with nodes far away in the network
- Each interaction costs

$$O\left(\sqrt{\frac{n}{\log n}}\right) = O\left(\frac{1}{r(n)}\right)$$

- But faster mixing
- Number of messages

$$T_{ave}(n) \sim O(n^{1.5})$$



# Path averaging

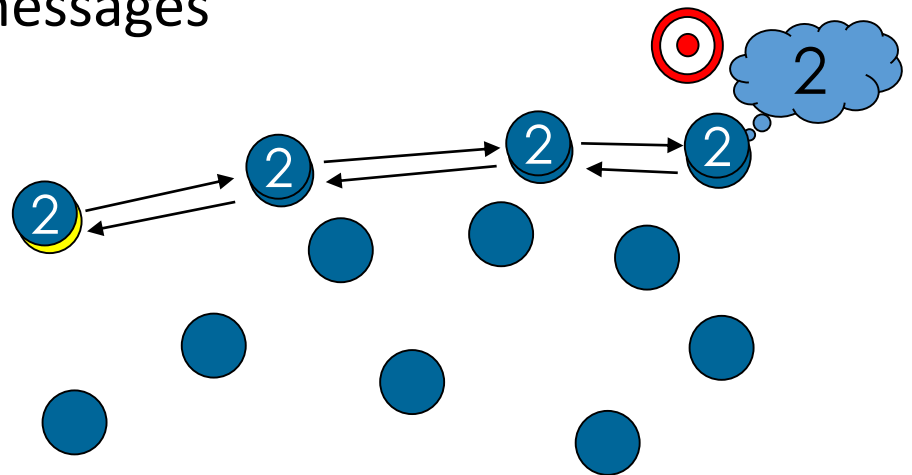
Averaging on the routed path?

The routed packet computes the sum of all the nodes it visits, and a hop-count. The average is propagated backwards to all the nodes on the path.

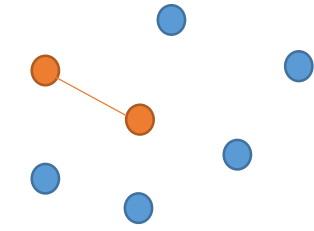
**Theorem:** Geographic gossip with path averaging on  $G(n, r)$  requires expected number of messages

$$T_{\text{ave}} = \Theta(n \log 1/\epsilon)$$

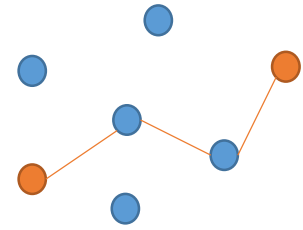
**Optimal** number of messages



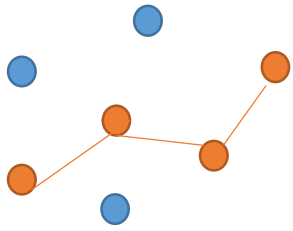
# Communication models



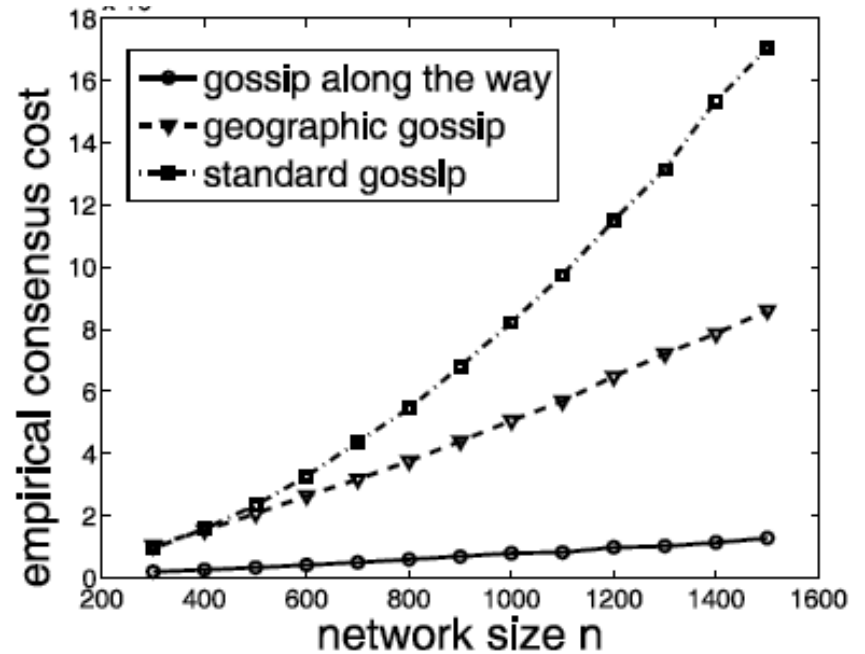
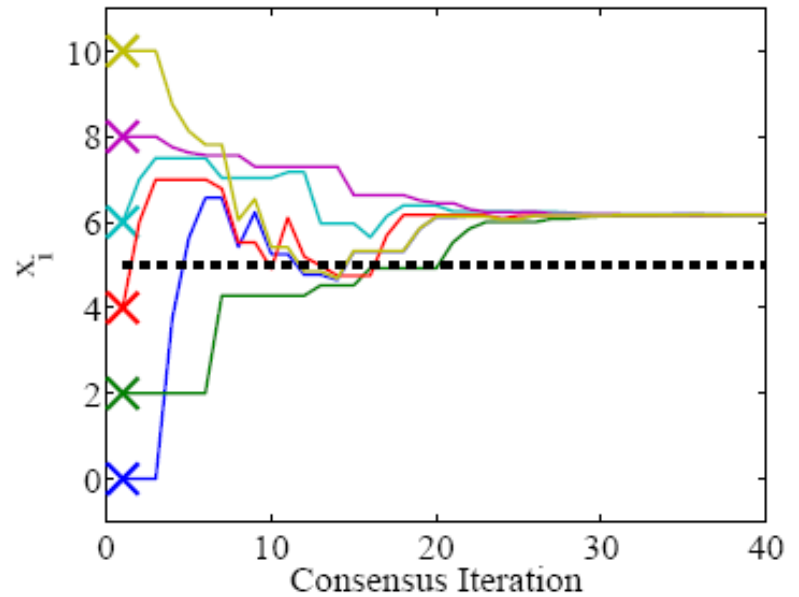
Pair-wise  
 $\Theta(n^2)$



Geographic routing  
 $\Theta(n^{1.5} \sqrt{\log n})$



Path averaging  
 $n \log n$



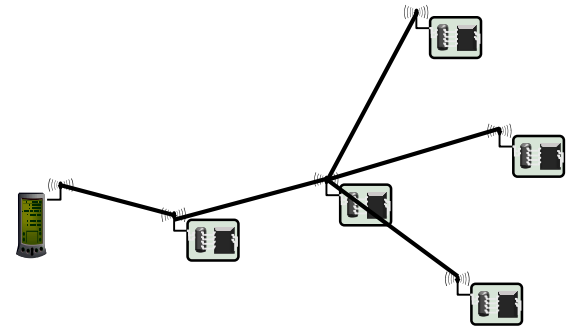
# Data management in WSN

## Conventional approach

Sample -> aggregate to sink -> perform analysis

Limitations

- Inefficient for large scale network
- Deployment constraints -> inaccessible sink



## Alternative approach

Nodes store data locally -> collector (mobile) gathers

Unreliable & failed nodes

Persistent data storage

# Distributed Data Storage

$n$ : nodes in the network

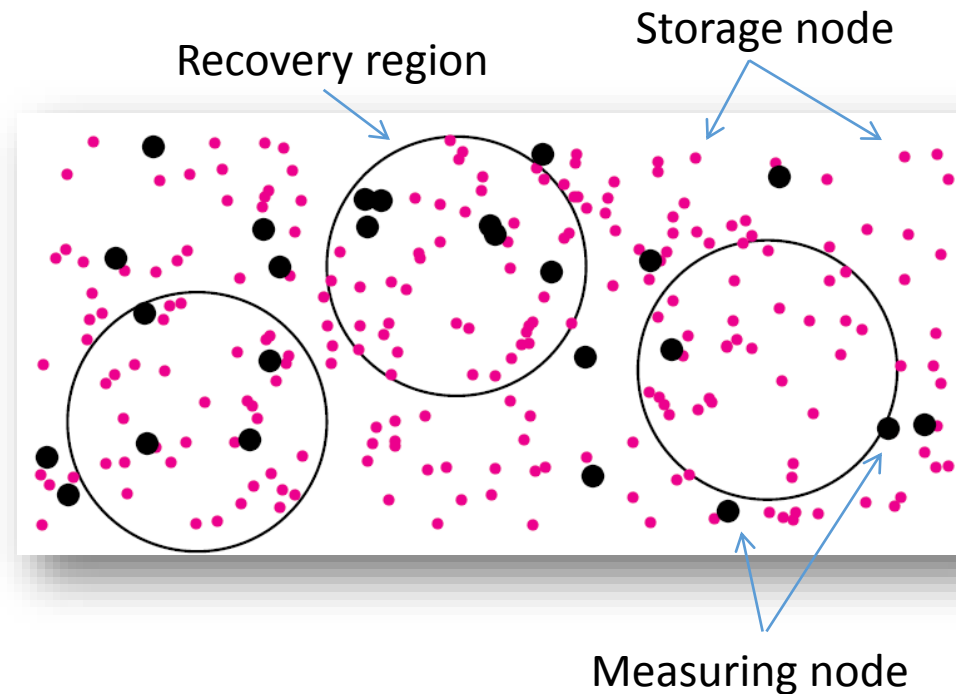
$k$ : sensors take measurements

## Objective

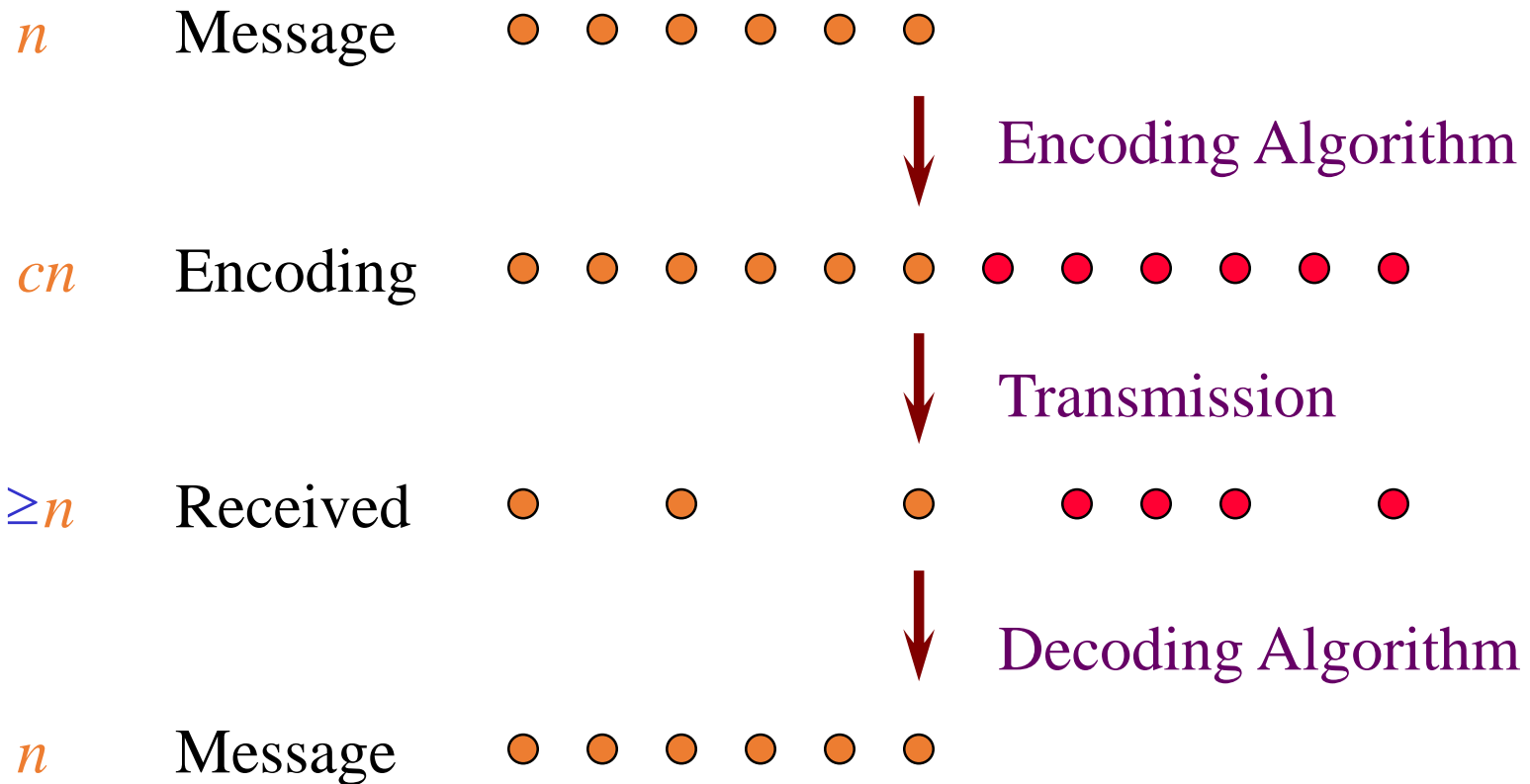
- Each sensors stores *one* packet
- Recovery from any  $k(1+\epsilon)$  nodes
- Decentralized operation

## Motivation

- Localized data gathering (energy)
- Recovery from failing networks



# Erasure Codes



# Data persistence with Fountain Codes

Erasur codes: encode message of  $k$  symbols  $\rightarrow$   $n$  symbols (where  $k < n$ ) s.t. original message can be recovered from a subset of the  $n$  symbols.

## **Fountain codes (rateless erasure codes)**

- limitless sequence of symbols  
from a given set of source symbols
- original source symbols can be recovered from *any subset* of the symbols  
of size *equal* to the number of source symbols
- Key representatives: **Luby Transform (LT)** and **Raptor**



# Erasure Codes: LT-Codes

1. Pick *degree*  $d_1$  from a pre-specified distribution. ( $d_1=2$ )
2. Select  $d_1$  input blocks uniformly at random. (Pick  $b_1$  and  $b_4$  )
3. Compute their sum (XOR).
4. Output sum, block IDs

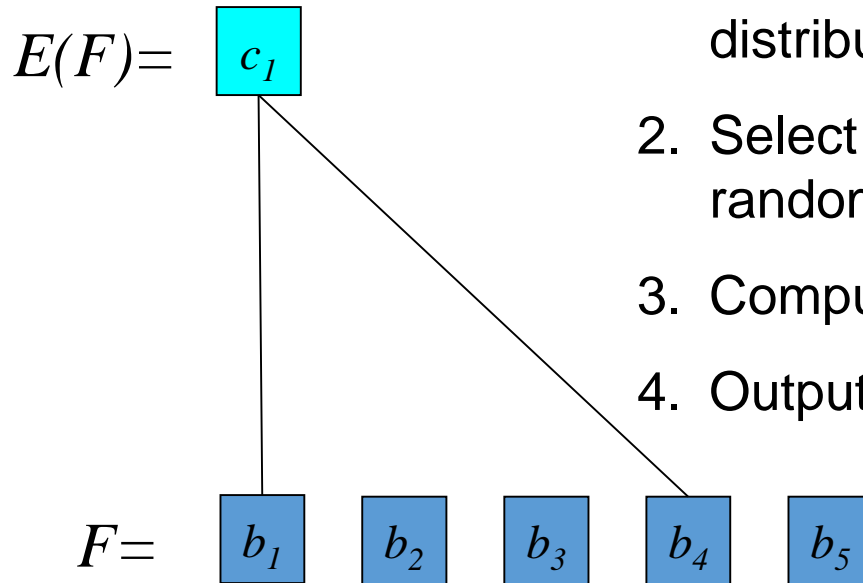
$$F = \begin{array}{|c|} \hline b_1 \\ \hline \end{array} \begin{array}{|c|} \hline b_2 \\ \hline \end{array} \begin{array}{|c|} \hline b_3 \\ \hline \end{array} \begin{array}{|c|} \hline b_4 \\ \hline \end{array} \begin{array}{|c|} \hline b_5 \\ \hline \end{array}$$

$n=5$  input blocks





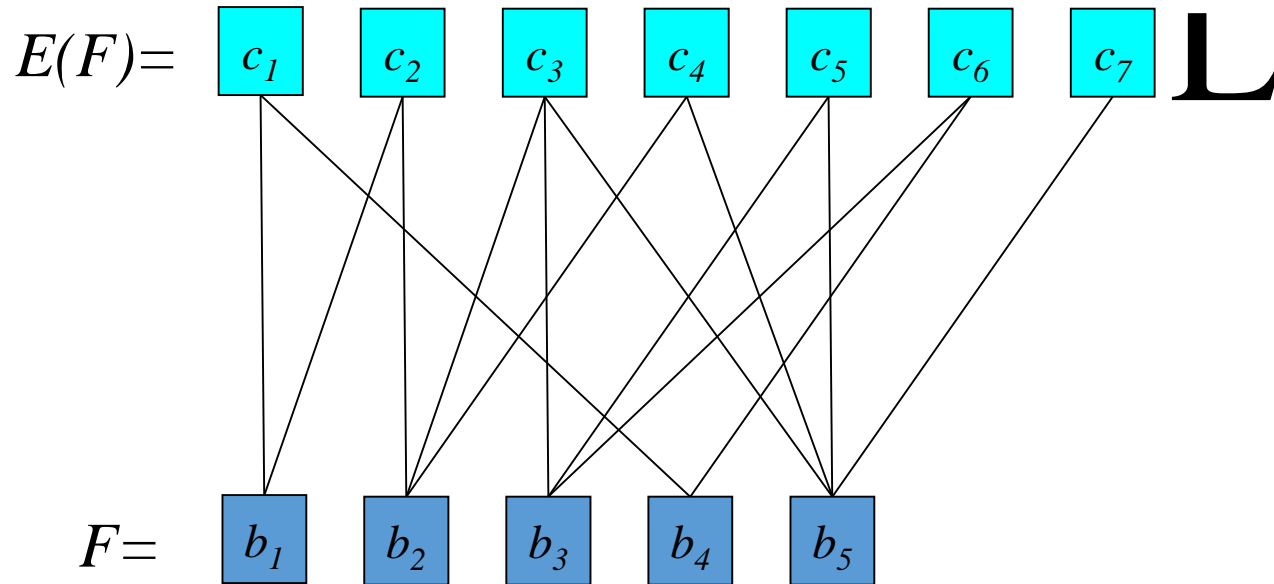
# LT-Codes: Encoding



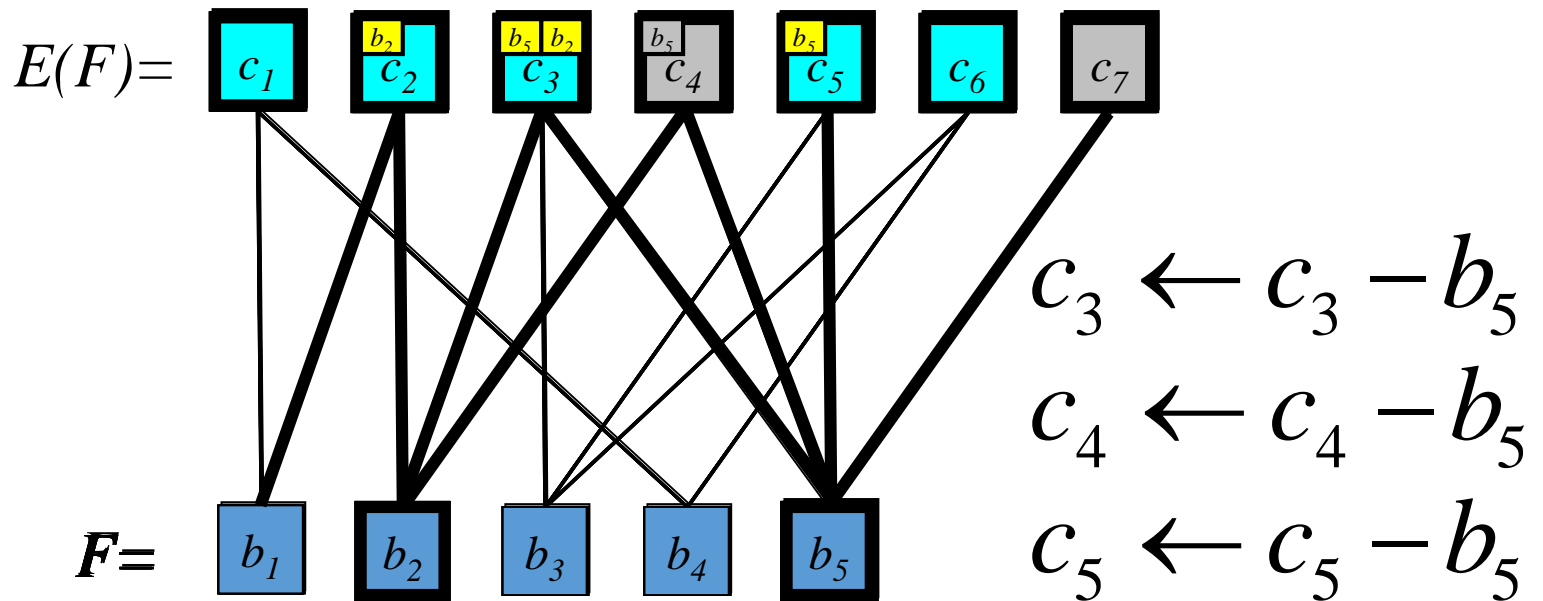
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# LT-Codes: Encoding



# LT-Codes: Decoding



# Linear code fundamentals

- Generator matrix  $\mathbf{s}=\mathbf{mG}$
- $\mathbf{s}$  = encoded vector,  $\mathbf{m}$  = input vector,  $\mathbf{G}$  in  $\mathbb{R}^{M \times K}$
- $K$  = degree

Ideal Soliton distribution

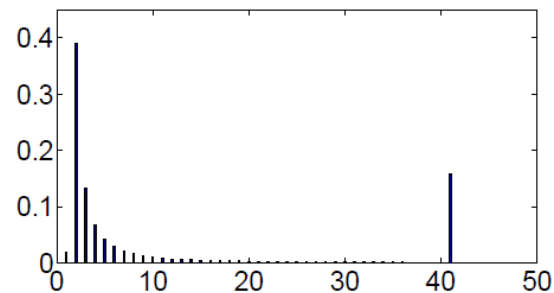
$$\rho(i) = \begin{cases} 1/K & \text{if } i = 1, \\ 1/i(i-1) & \text{for } i = 2, 3, \dots, K. \end{cases}$$

Recovery

- $k + O(\sqrt{k} \ln^2(k/\delta))$  encoding symbols w.h.p.  $(1 - \delta)$ .

Complexity

- $O(k \ln(k/\delta))$

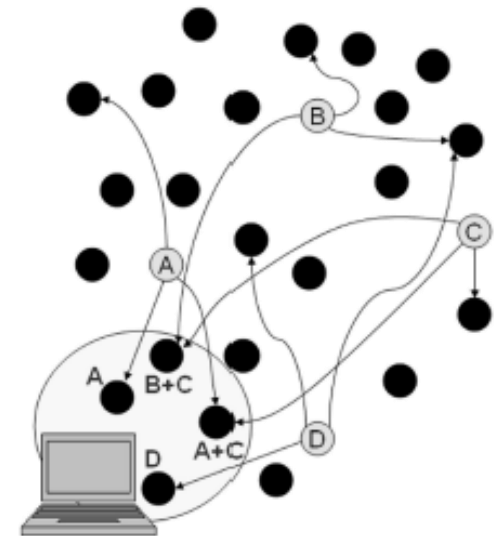


# Data dissemination

- A random walk with length  $L$  will stop at a node.
- If the length  $L$  of random walk is sufficiently long, then the distribution will achieve steady state.

## Algorithmic steps

- Step 1 : Degree generation
- Step 2 : Compute steady-state distribution
- Step 3 : Compute probabilistic forwarding table
- Step 4 : Compute the number of random walks
- Step 5 : Block dissemination
- Step 6: Encoding



# Distributed Data Storage with LC

## Encoding and Storage Phase (at all nodes $u$ )

- 1) Node  $u$  draws  $d_c(u)$  from  $\{1, \dots, \hat{k}(u)\}$  according to  $\Omega$ .
- 2) Upon receiving packet  $x$ , if  $c(x) < C_1 \hat{n} \log \hat{n}$ , node  $u$ 
  - puts  $x$  into its forward queue and increments  $c(x)$ .
  - with probability  $d_c(u)/\hat{k}$ , accepts  $x$  for storage and updates its storage variable  $y_u^-$  to  $y_u^+$  as

$$y_u^+ = y_u^- \oplus x_s, \quad (14)$$

If  $c(x) < C_1 \hat{n} \log \hat{n}$ ,  $x$  is removed from circulation.

- 3) When a node receives a packet before the current round, it forwards its head-of-line (HOL) packet to a randomly chosen neighbor.
- 4) Encoding phase ends and storage phase begins when each node has seen its  $\hat{k}(u)$  source packets.



# Experimental results

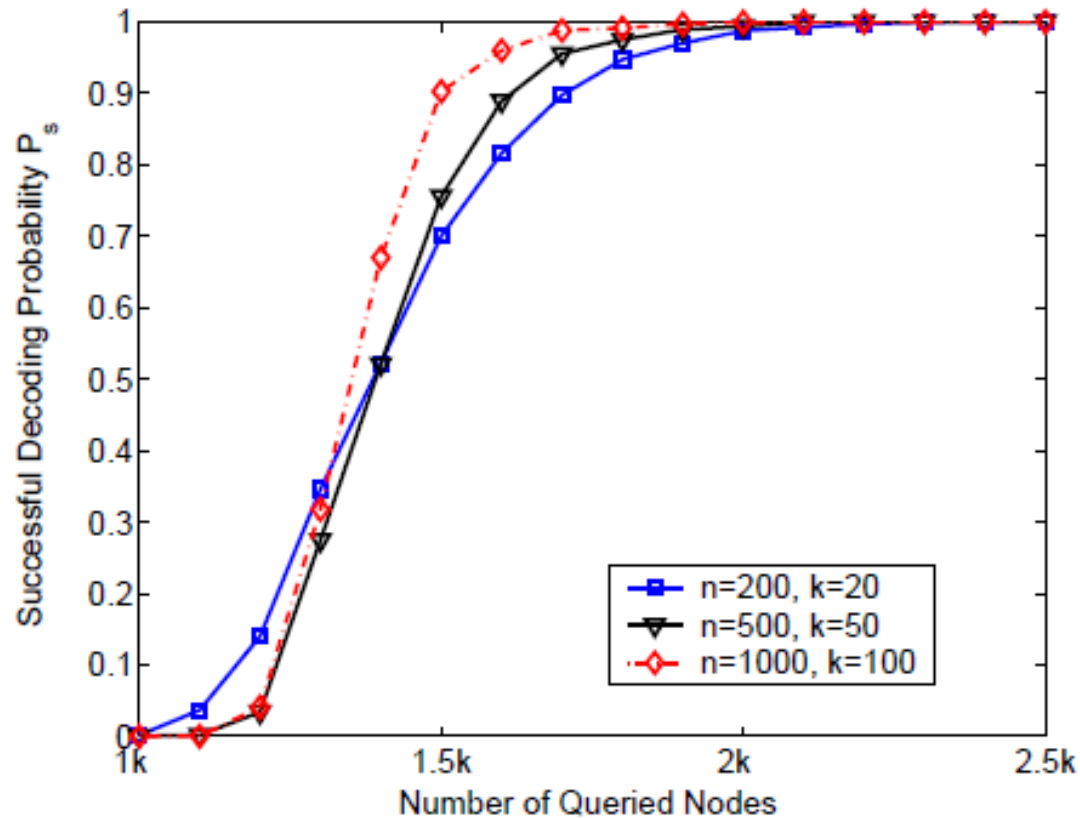
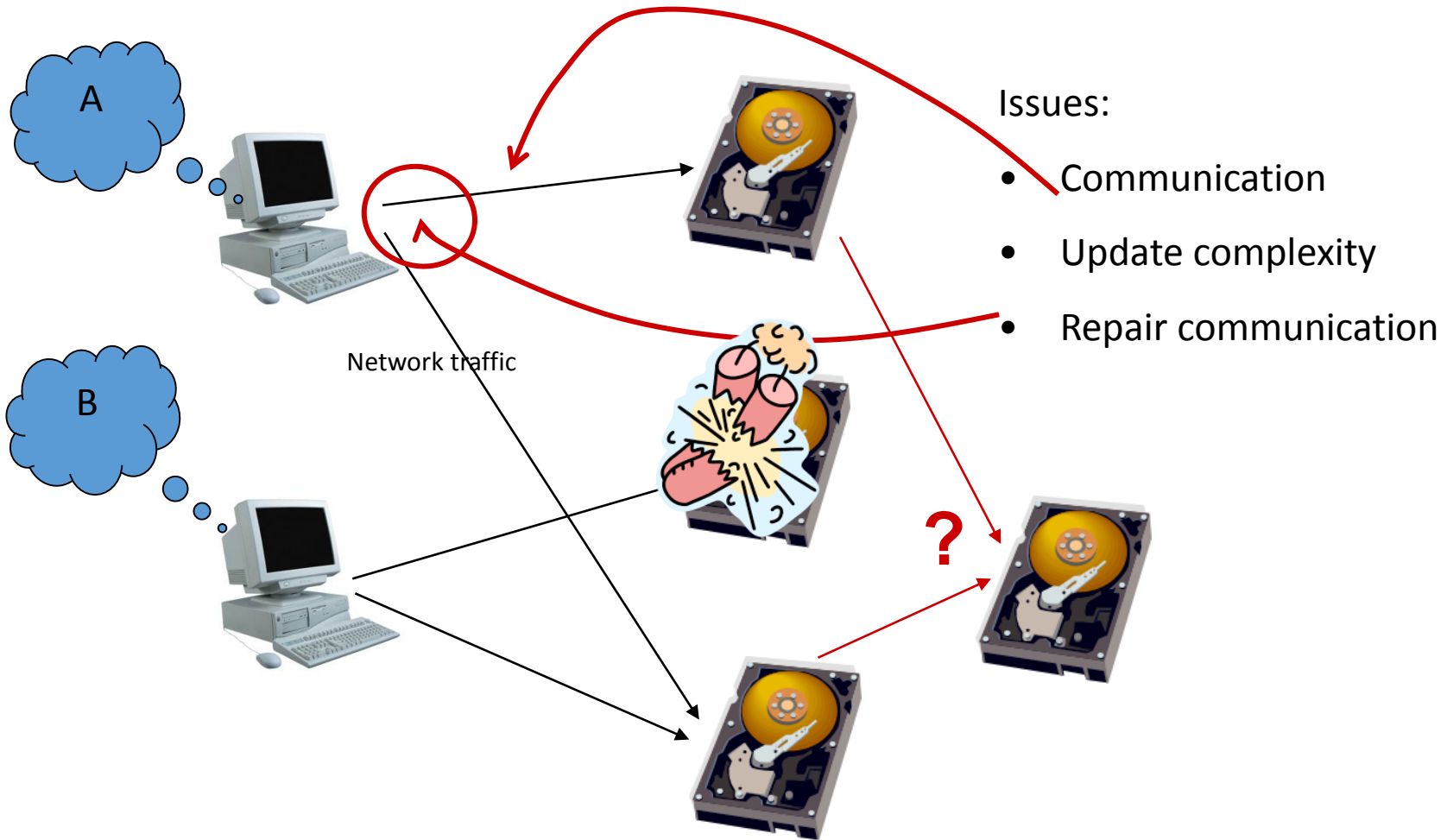


Fig. 2. Performance of LTCDS with known  $n$  and  $k$  for (a)  $n=200, k=20$ ; (b)  $n=500, k=50$ ; and (c)  $n=1000, k=100$ .

# Coding + Storage Networks = New open problems





# Reading List

- Dimakis, Alexandros G., et al. "**Gossip algorithms for distributed signal processing.**" *Proceedings of the IEEE* 98.11 (2010): 1847-1864.
- Shuman, David I., et al. "**The emerging field of signal processing on graphs: Extending high-dimensional data analysis to networks and other irregular domains.**" *Signal Processing Magazine, IEEE* 30.3 (2013): 83-98.
- Ostovari, Pouya, Jie Wu, and Abdallah Khreishah. "**Network coding techniques for wireless and sensor networks.**" *The Art of Wireless Sensor Networks*. Springer Berlin Heidelberg, 2014. 129-162.
- Guide to Wireless Sensor Networks eds S. Misra, I. Woungang, S. C. Misra, Chapter 7: Data-Centricity in Wireless Sensor Networks, Abdul-Halim Jallad and Tanya Vladimirova.
- Dimakis, Alexandros G., et al. "A survey on network codes for distributed storage." *Proceedings of the IEEE* 99.3 (2011): 476-489.

