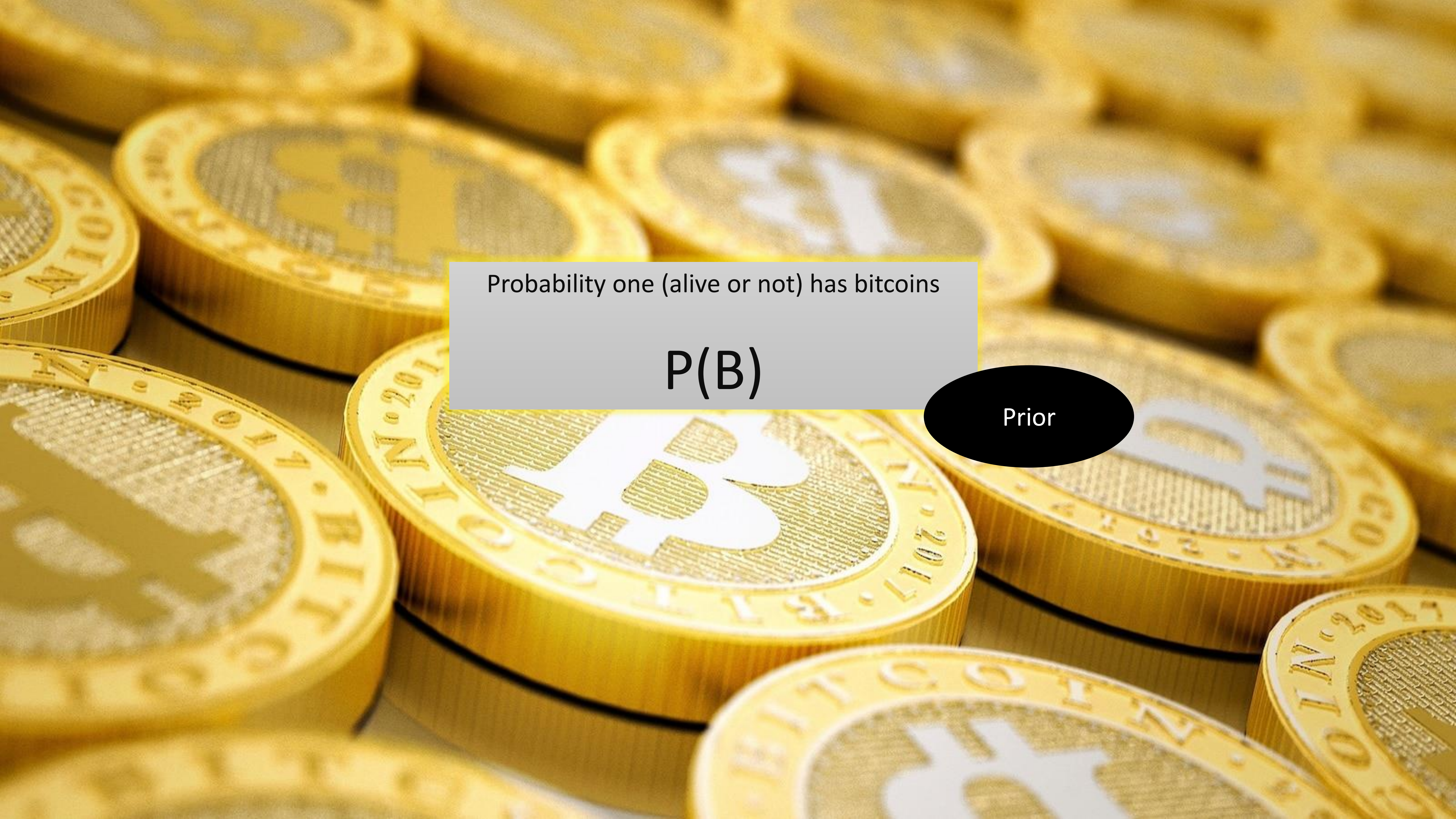
- Terms & Definitions
- Bayes Theorem - Simple Example
- Naïve Bayes Classifier – SOS Example
- Q. Assignment 1

Tutorial 1
CS-473



Probability one (alive or not) has bitcoins

P(B)

Prior

Probability one (alive or not) has bitcoins

in **2025**

$$P(B|Y=\mathbf{2025})$$

Posterior

Why?

P(A): Cloudy
P(B): Raining

P(B|A): Raining given day is cloudy
P(A|B): Cloudy given it is raining

Bayes' Theorem EXPLAINED with Examples

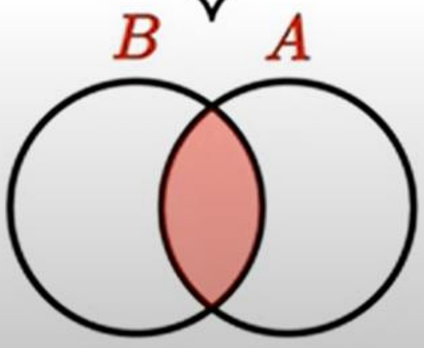
Bayes' Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$$P(B \cap A)$$

$$\Downarrow$$



$$P(A) \cdot P(B|A) = \frac{P(A \cap B)}{\cancel{P(A)}} \cdot \cancel{P(A)}$$

$$P(B|A) \cdot P(A) = P(A \cap B)$$

$$P(A \cap B) = P(B \cap A)$$

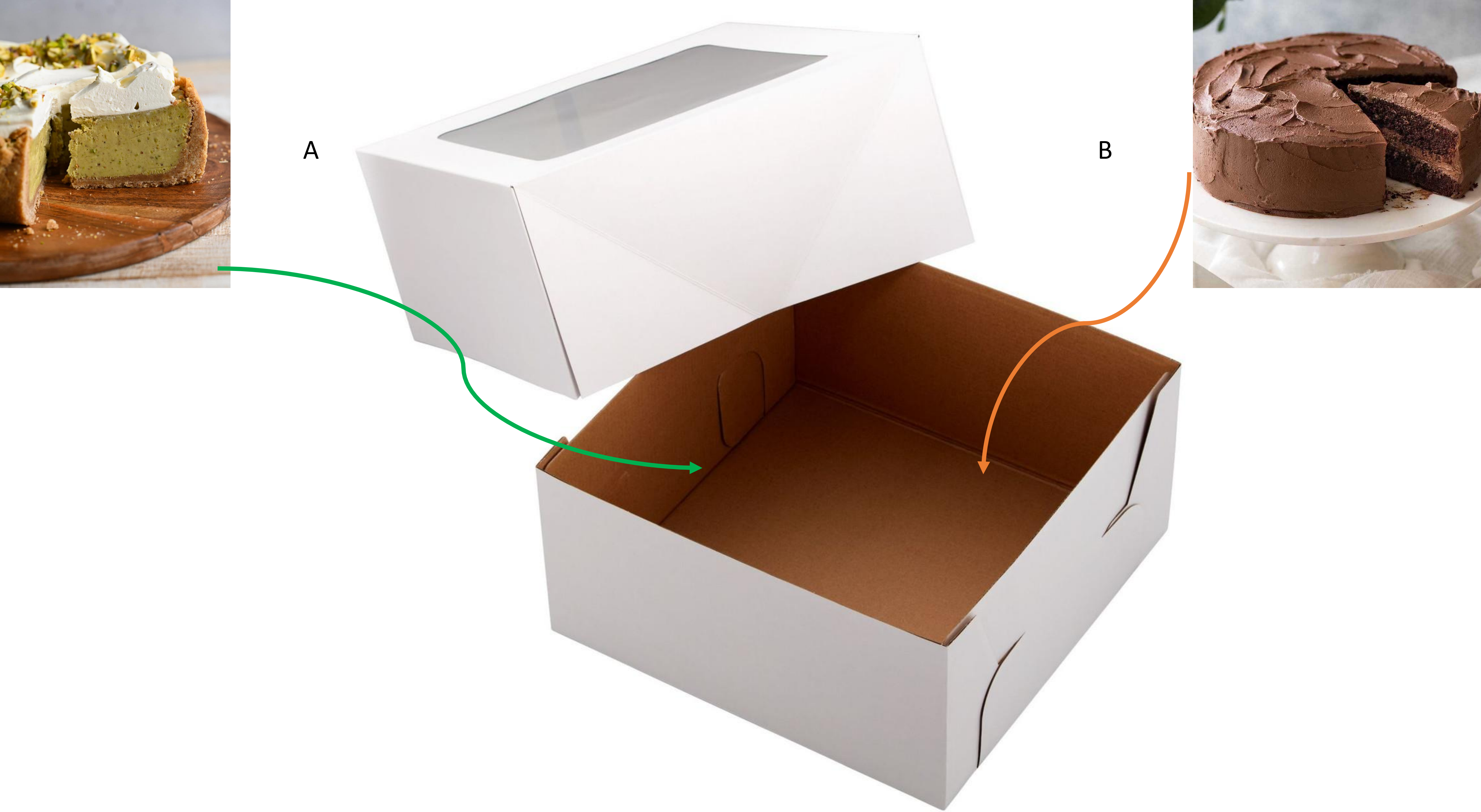
https://www.youtube.com/watch?v=cqTwHnNbc8g

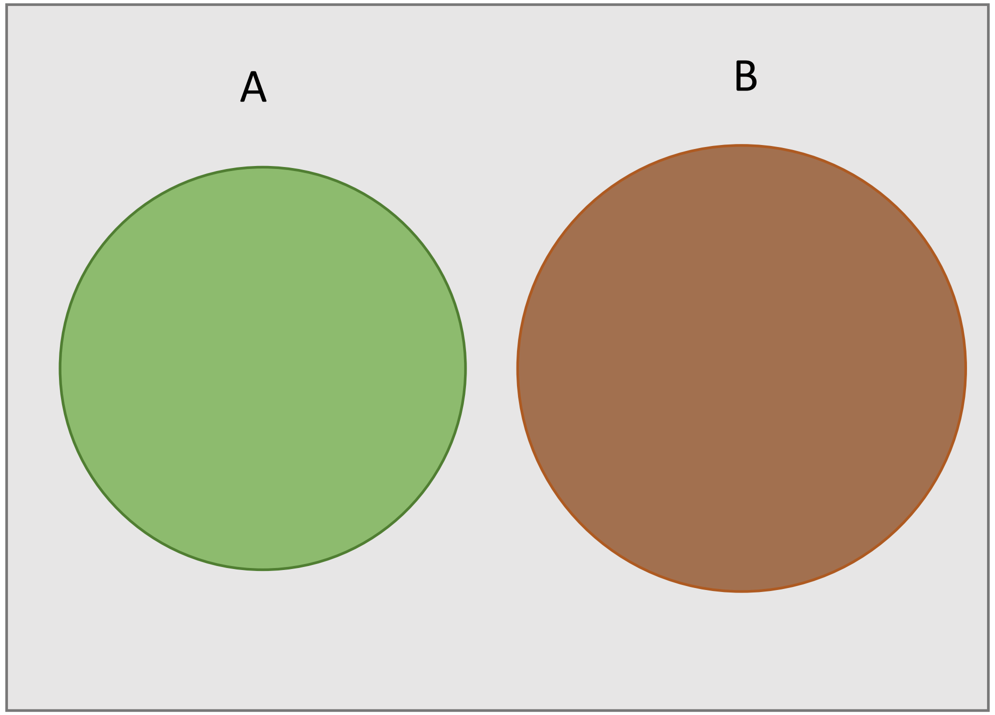
4:01 / 8:02

Cakes

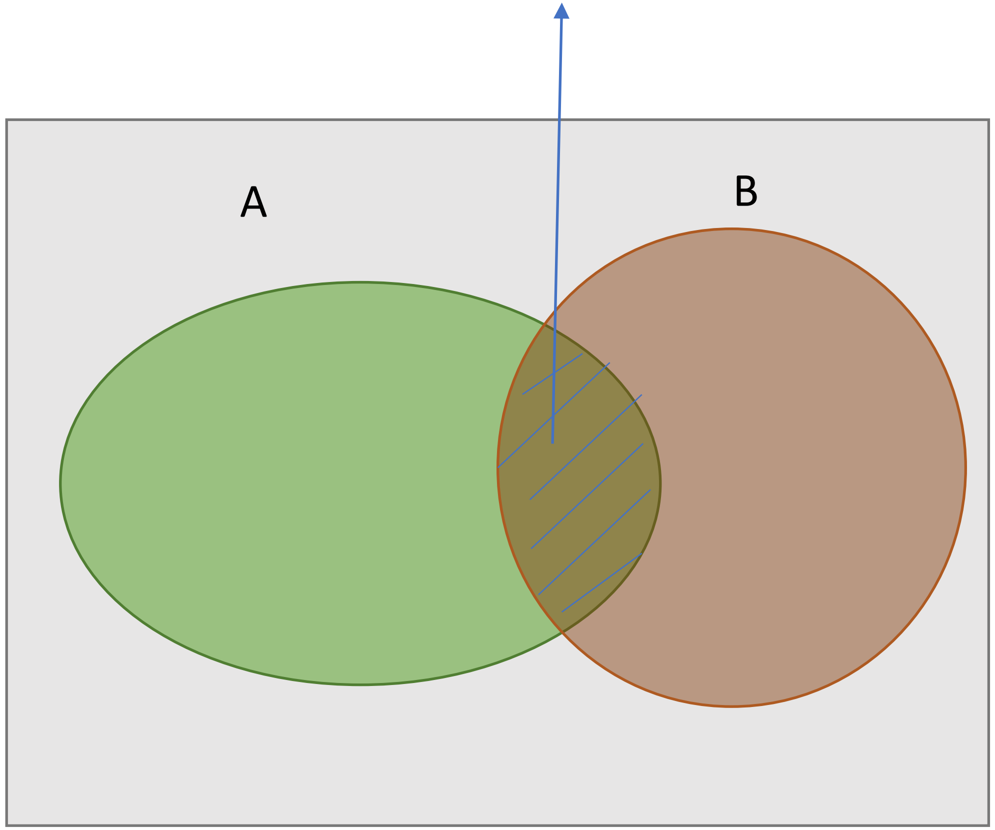
A

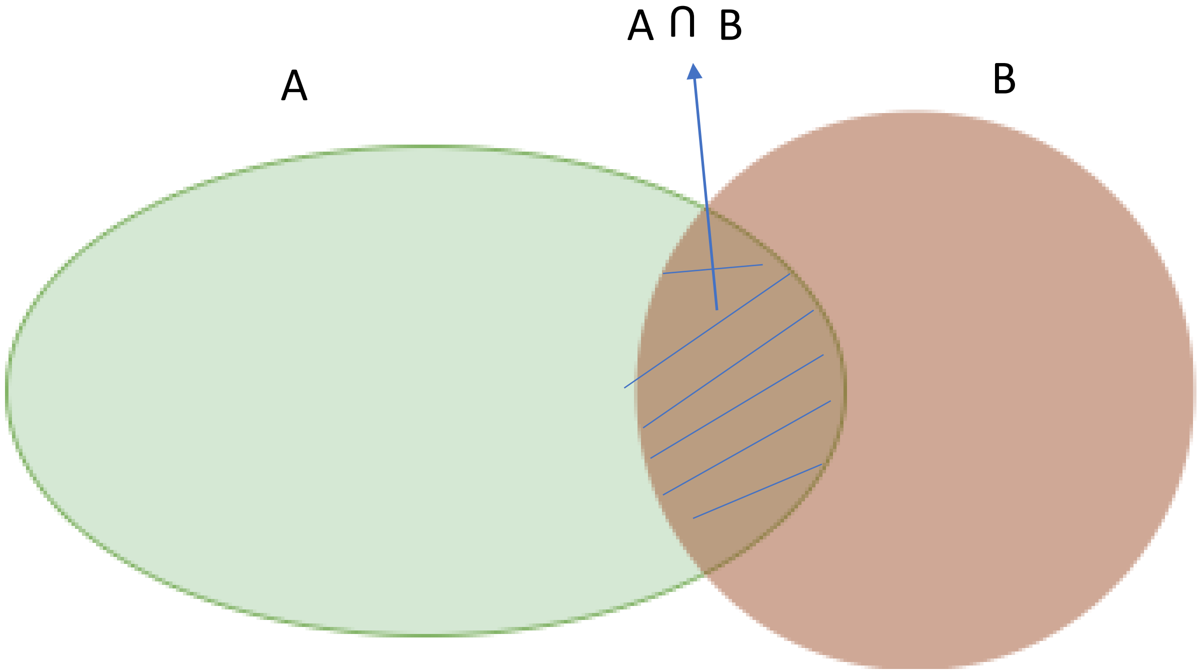
B


A
B

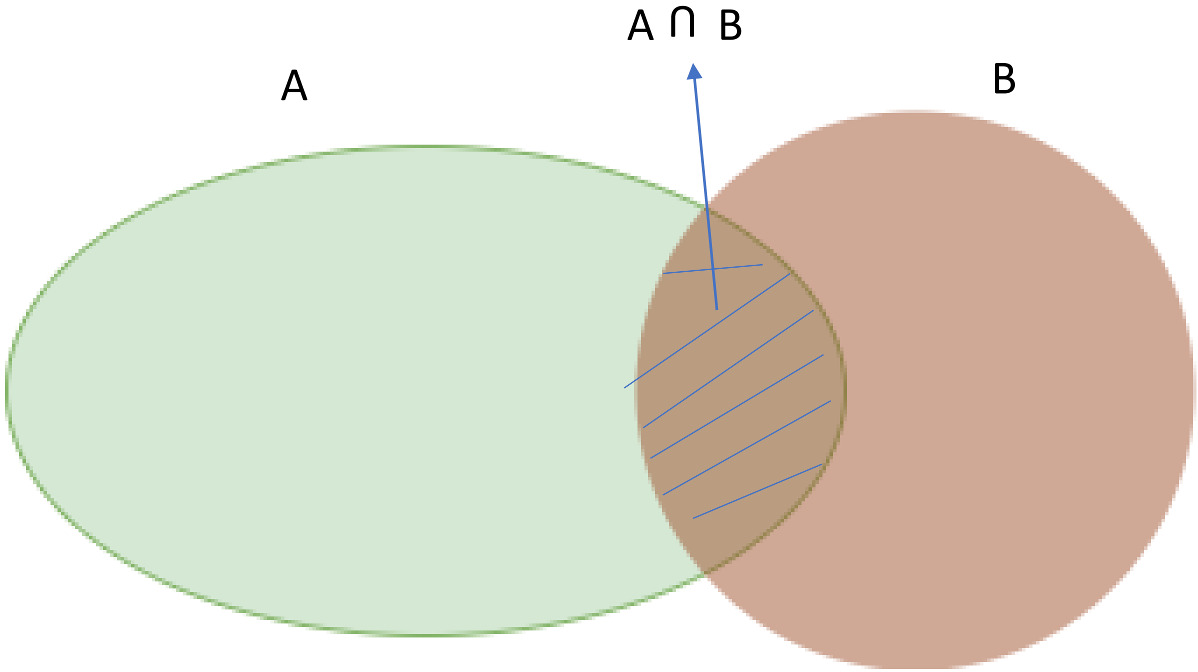

A
B

$A \cap B$




A ∩ B
A
B

A | B ?



A

A | B

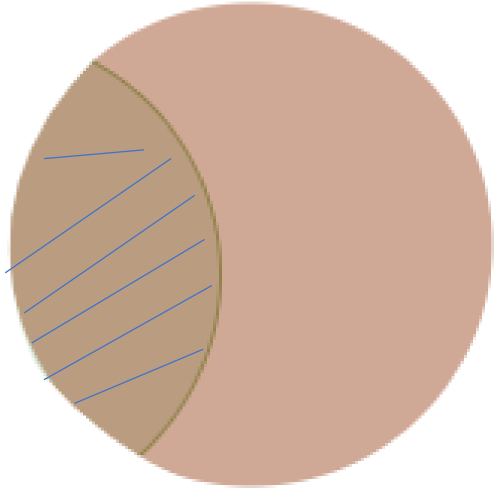
A ∩ B

?

B

P(B)

P(A|B)



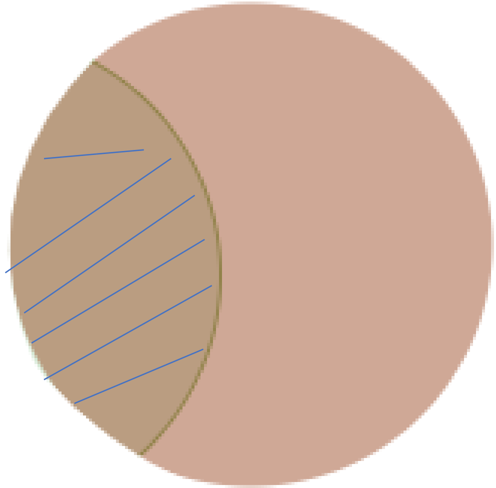
A

$$P(A|B)=P(A \cap B) / P(B)$$

B

P(B)

P(A|B)



Gaussian Naive Bayes is a machine learning **classification technique** based on a probablistic approach that assumes each class follows a **normal distribution**.

It assumes each parameter has an **independent** capacity of predicting the output variable.

Gaussian Naive Bayes Classifier for Continuous Data

- Continuous values associated with each feature are assumed to be distributed based on Gaussian distribution.
- Likelihood of the features is assumed to be Gaussian.

Gender	Height (ft)	Weight (lbs)	Foot size (inch)
Male	6	180	12
Male	5.92	190	11
Male	5.58	170	12
Male	5.92	165	10
Female	5	100	6
Female	5.5	150	8
Female	5.42	130	7
Female	5.75	150	9

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- Example:
- Classify whether a given person's datapoint is a male or a female.
- The features include height, weight, and foot size.

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- Classify whether a given person's datapoint is a male or a female.
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- P(Male) = 4/8 = 0.5 ; P(Female) = 4/8 = 0.5

Male:

- Mean(height) = 5.855
- Variance(Height) = $\frac{\sum(x_i-\overline{x})^2}{n-1} = \frac{(6-5.855)^2+(5.92-5.855)^2+(5.58-5.855)^2+(5.92-5.855)^2}{4-1}$ =0.035055
- Calculate for all features

Gender	Mean (height)	Variance (Height)	Mean (weight)	Variance (Weight)	Mean (Footsize)	Variance (Footsize)
Male	5.855	0.035	176.25	122.92	11.25	0.91667
Female	5.4175	0.097225	132.5	558.33	7.5	1.6667

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Gaussian Naive Bayes Classifier for Continuous Data

- Continuous values associated with each feature are assumed to be distributed based on Gaussian distribution.
- Likelihood of the features is assumed to be Gaussian.
- So, conditional probability is given by: $P(x_i|y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} e^{-\frac{(x_i-\mu_y)^2}{2\sigma_y^2}}$

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- Classify whether a given person's datapoint is a male or a female.
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- P(Male) = 4/8 = 0.5 ; P(Female) = 4/8 = 0.5

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Gaussian Naive Bayes Classifier for Continuous Data

- Classify the new datapoint

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- $P(\text{male}) = \dfrac{P(\text{male}) * P(\text{H}|\text{male})\, P(\text{w}\,|\text{male})\, P(\text{F}|\text{male})}{\text{Marginal probability or Evidence}}$

- $P(\text{female}) = \dfrac{P(\text{female}) * P(\text{H}|\text{female})\, P(\text{w}\,|\text{female})\, P(\text{F}|\text{female})}{\text{Marginal probability or Evidence}}$

$$P(x_i|y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} e^{-\frac{(x_i - \mu_y)^2}{2\sigma_y^2}}$$

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- Classify the new datapoint

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- $\text{P(male)} = \dfrac{\text{P(male)} * \text{P(H|male) P(W|male) P(F|male)}}{\text{Marginal probability or Evidence}}$
- $\text{P(female)} = \dfrac{\text{P(female)} * \text{P(H|female) P(W|female) P(F|female)}}{\text{Marginal probability or Evidence}}$
- The evidence (normalizing constant) is the sum of the posteriors equals one.
- evidence=P(male)*P(ht|male)*P(wt|male)*P(footsize|male)+P(female)*P(ht|female)*P(wt|female)*P(foot size|female)
- The evidence may be ignored since it is a positive constant. (Normal distributions are always positive)

$$P(x_i|y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} e^{-\frac{(x_i - \mu_y)^2}{2\sigma_y^2}}$$

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- The evidence may be ignored since it is a positive constant. (Normal distributions are always positive)

- $P(H|M) = \frac{1}{\sqrt{2*3.14*0.035033}} * e^{-\frac{(6-5.855)^2}{2*0.035033}}$ =1.5789 P(H|F) = 2.2356e^{-1}
- P(W|M) = 5.9881e^{-6} P(W|F)=1.6789e^{-2}
- P(Foot|M) = 1.3112e^{-3} P(Foot|F) = 2.8669e^{-1}

- $\text{P(male)} = \frac{\text{P(male)} * \text{P(H|male) P(W|male) P(F|male)}}{\text{Marginal probability or Evidence}}$=0.5*1.5789*5.9881e^{-6}*1.3112e^{-3}=6.1984e-9
- $\text{P(female)} = \frac{\text{P(female)} * \text{P(H|female) P(W|female) P(F|female)}}{\text{Marginal probability or Evidence}}$=0.5*2.2346e^{-1}*1.6789e^{-2}*2.8669e^{-1}=**5.377e^{-4}**
- **P(female)** > P(male)