Δ.10 $R = \{(x,y): 0\le x\le 1,\ 2\le y\le 3\}$

$$\iint_R f(x,y)\,dx\,dy = \int_0^1\int_2^3 f(x,y)\,dy\,dx =$$

$$\int_0^1\left(\int_2^3 x^2+y\,dy\right)dx = \int_0^1\left[\frac{y^2}{2}+x^2\cdot y\right]_{y=2}^{y=3}dx$$

$$=\int_0^1\left(x^2+\frac{5}{2}\right)dx = \left[\frac{x^3}{3}+\frac{5}{2}x\right]_{x=0}^{x=1} = \frac{1}{3}+\frac{5}{2}$$

$$= \frac{17}{6} = \int_2^3\int_0^1 f(x,y)\,dx\,dy$$

Δ.13 $\int_0^1\int_0^{x^2} x^2+y+1\,dy\,dx =$

$$=\int_0^1 x^2\cdot y+\frac{y^2}{2}+y\,\Big|_{y=0}^{y=x^2}dx=$$

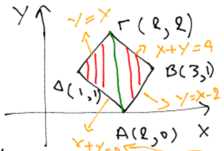
$$=\int_0^1 \frac{3x^4}{2}+x^2\,dx = \frac{3}{10}x^5+\frac{x^3}{3}\Big|_{x=0}^{x=1}$$

$$=\frac{3}{10}+\frac{1}{3}=\frac{19}{30}$$

Άσκηση

$f(x,y)=x^2+y^2$

R: ρόμβος ΑΒΓΔ



ΑΔ: $\langle x(t), y(t)\rangle = \langle 2,0\rangle + t\cdot\langle -1,1\rangle$

$$R = A\Delta\Gamma + AB\Gamma = \left\{\begin{array}{l}2-x\le y\le x\\ 1\le x\le 2\end{array}\right\} + \left\{\begin{array}{l}x-2\le y\le 4-x\\ 2\le x\le 3\end{array}\right\}$$

$$\begin{cases}x(t)=2-t\\ y(t)=t\end{cases}\Rightarrow \boxed{x=2-y}$$

$$\iint_R f(x,y)\,dx\,dy =$$

$$\int_1^2\int_{2-x}^{x} f(x,y)\,dy\,dx +$$

$$\int_2^3\int_{x-2}^{4-x} f(x,y)\,dy\,dx$$

Δ.30 $\int_1^3\int_0^2\left(\frac{1}{2}+2y+z\right)dy\,dz =$

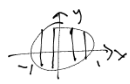
$$=\int_1^3(1+4+2z)\,dz=$$

$$=5z+\frac{2z^2}{2}\Big|_{z=1}^{z=3} = 15+9-(5+1) = 18$$

$$2\cdot 2\cdot 2\cdot\left(\frac{1}{2}+2\cdot 1+2\right) = 4\cdot 4{,}5 = 18$$

Δ.44

$$R=\left\{\begin{array}{l}0\le z\le\sqrt{1-x^2-y^2}\\ -\sqrt{1-x^2}\le y\le\sqrt{1-x^2}\\ -1\le x\le 1\end{array}\right\}\ (z=0)\to x^2+y^2=1$$



$$\int_{-1}^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} 1\,dz\,dy\,dx$$

$$=\int_{-1}^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\sqrt{1-x^2-y^2}\,dy\,dx = ?$$

$$f_1(r,\vartheta) = \sqrt{1-r^2}$$

$$R = \left\{ \begin{array}{l} 0 \le r \le 1 \\ 0 \le \theta \le 2\pi \end{array} \right\}$$

$$\int_0^{2\pi}\int_0^1 \sqrt{1-r^2}\cdot r\,dr\,d\vartheta = -\int_0^{2\pi} \frac{(1-r^2)^{3/2}}{3/2}\cdot\frac{1}{2}\Big|_{r=0}^{r=1} d\vartheta$$

$$= -\int_0^{2\pi}\left(0-\frac{1}{3}\right)d\vartheta = \frac{2\pi}{3}$$

Δ.45

$$\int_0^{2\pi} \frac{e^{r^2}}{2}\Big|_{r=0}^{r=1} d\vartheta = \int_0^{2\pi} \frac{e-1}{2}\,d\vartheta =$$

$$= (e-1)\cdot\pi$$